IX. *An Analysis of the Results from the Kew Magnetographs on "Quiet" Days during the Eleven Years* 1890 *to* 1900, *with a Discussion of Certain Phenomena in the Absolute Observations.*

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(*From the National Physical Laboratory.*)

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Contents.

§ 1. In 1889 the then Kew Committee agreed to take part in a scheme which provided that at the end of each year the Astronomer Royal should select five magnetically *quiet* days for each month with a view to the tabulation of their records. The scheme has been in continuous operation since 1890, and the results from the selected quiet days have been published annually in the 'Report' made to the Royal Society.

In 1895 I contributed to the British Association 'Report' an analysis of the declination and horizontal force records for the years 1890 to 1894, and described the previously unnoticed "non-cyclic effects" which appear to be characteristic of magnetically quiet days at Kew and elsewhere. In 1901 the development of electric traction in the West of London rendered it clear that no further records at Kew would be free from suspicion of artificial disturbances, and that it would be necessary in the near future to make arrangements for continuing the magnetic work elsewhere. Both contingencies pointed to the expediency of a complete discussion of the data obtained from 1890 to 1900, which I accordingly commenced. The delay in the execution of the scheme is largely due to the occurrence of various unforeseen difficulties, which have, however, I think been at last fairly satisfactorily surmounted. The object of the paper is not so much to put observations on record, as to make a critical use of them. The data used are mainly derived from old Kew 'Reports,' to which reference must be made for particulars.

§ 2. One of the questions dealt with is the relation between sun-spots and terrestrial magnetism. The sun-spot data made use of are from the very important table published by WOLFER in the 'Met. Zeit.'* WOLFER's table gives the mean monthly values assigned to the sun-spot frequency by WOLF and WOLFER for a very long series of years. As frequent reference has to be made to sun-spot data throughout the whole of this paper, it is convenient to reproduce at once that part of WOLFER's table which applies to the period 1890 to 1900 now under review. This will be found in Table I. In addition to WOLFER's own figures I give the mean sun-spot frequency obtained for each month of the year when we combine the 11 years 1890 to 1900.

It will be noticed that the four years 1892 to 1895 are conspicuously years of sun-spot maximum, while 1890, 1899, and 1900 are years of very few sun-spots. The mean frequency for the 12 months commencing August, 1893, and ending July, 1894, is higher than for any other 12 successive months in the 11-year period. Considering that so many as 11 years are included, the irregularity in the mean values for the several months of the year is rather surprising. March and November in particular show a remarkably low mean frequency. This is to be regretted, as it introduces a certain element of uncertainty into some of the calculations made in the latter part of the paper.

* May, 1902, p. 193, *et seq.*

TABLE I.—Sun-Spot Frequency (after WOLFER).

	1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.	1898.	1899.	1900.	Means for each month of the year.
January .	5·3	13·5	69·1	75·0	83·2	63·3	29·0	40·6	30·2	19·5	9·4	39·8
February .	0·6	22·2	75·6	73·0	84·6	67·2	57·4	29·4	36·4	9·2	13·6	42·65
March . .	5·1	10·4	49·9	65·7	52·3	61·0	52·0	29·1	38·3	18·1	8·6	35·5
April . .	1·6	20·5	69·6	88·1	81·6	76·9	43·8	31·0	14·5	14·2	16·0	41·6
May . .	4·8	41·1	79·6	84·7	101·2	67·5	27·7	20·0	25·8	7·7	15·2	43·2
June . .	1·3	48·3	76·3	88·2	98·9	71·5	49·0	11·3	22·3	20·5	12·1	45·4
July . .	11·6	58·8	76·8	88·8	106·0	47·8	45·0	27·6	9·0	13·5	8·3	44·8
August .	8·5	33·2	101·4	129·2	70·3	68·9	27·2	21·8	31·4	2·9	4·3	45·4
September	17·2	53·8	62·8	77·9	65·9	57·7	61·3	48·1	34·8	8·4	8·3	45·1
October .	11·2	51·5	70·5	79·7	75·5	67·9	28·4	14·3	34·4	13·0	12·9	41·75
November	9·6	41·9	65·4	75·1	56·6	47·2	38·0	8·4	30·9	7·8	4·5	35·0
December.	7·8	32·2	78·6	93·8	60·0	70·7	42·6	33·3	12·6	10·5	0·3	40·2
Means for individual years . .	7·1	35·6	73·0	84·9	78·0	64·0	41·8	26·2	26·7	12·1	9·5	41·72

Secular Change.

§ 3. Few people at all appreciate the difficulty of arriving at anything approaching certainty as to the true value of the secular change of magnetic elements from one year to the next. An examination of the published records of any two observatories, at such moderate distances apart as Potsdam and Parc St. Maur (Paris), or even when so close together as Kew and Greenwich, discloses irregularities which are not shared by the two stations. In the present state of our knowledge, we can only say that such irregularities *may* be absolutely faithful representations of the operations of Nature, but personally I am inclined to the belief that they often owe a good deal to instrumental or similar causes. Building operations, changes of instruments, of observers, or of methods of reduction, are all apt to influence observational results, and of late years electric trams have been wrecking observatories, or introducing uncertainties into their records, all over the world. In the present case, this last and worst cause of uncertainty may be left out of account, but some of the others had, unfortunately, to be dealt with. Whenever it appeared necessary, I have referred to the original records, and have calculated so far as possible what the results would have been if the methods and instruments employed had been the same as of late years. To explain the situation, a brief account is necessary of the present method of deducing results from the measurement of the curves. To economise space, I shall throughout the rest of the paper distinguish the several magnetic elements by letters, and employ these, when convenient, in the text and tables. The letters so employed and their meanings are as follows:—

D = declination,	N = northerly component of H,
I = inclination,	W = westerly ,, ,,
H = horizontal force,	T = total force (resultant of H and V, or of N,
V = vertical force,	W and V).

§ 4. Taking first the case of D, as simplest, we have the scale value—that is the equivalent of 1 centim. of curve ordinate—dependent only on the distance between the photographic paper and the mirror carried by the declination magnet. There is no occasion to move this mirror except slightly in azimuth, as the magnet turns with secular change, and no appreciable change of scale value has occurred during the period dealt with. In this case no temperature compensation is necessary, so far as is known; and the scale value being determined once for all, there remains only to determine the value of the declination which answers to the base (or time) line, from which ordinates are measured. This is done by comparing the results of the absolute observations, taken usually three or four times a month, with the corresponding curve ordinates; each month is dealt with by itself. With the exception of a few months early in 1890, the same magnet has been used throughout for the absolute observations, and an elaborate intercomparison of it with the magnet previously accepted as the standard, before the latter was laid aside, gave mean results as nearly as possible identical. The magnetometer employed has been in use for about half a century. Certain additions were made to it in January, 1891, but were discarded after a few weeks' trial, and the results obtained with it during these weeks were rejected.

Coming next to H, the scale value here is affected by change either in the magnetograph magnet itself or in its bifilar suspension. During the whole eleven years the aim has been to keep the scale value at 1 centim. $= 50\gamma$ (where $1\gamma \equiv 1 \times 10^{-5}$ C.G.S. unit). The magnet is an old one, whose moment alters extremely slowly, and when the instrument is left to itself, the change of scale value taking place in twelve months seldom amounts to 1 per cent. As in the case of D, the value of the base line is determined by comparing absolute values of H, determined three or four times a month, with the corresponding curve ordinates, and each month is treated independently. A temperature correction is applied, which allows for the difference between the temperature of the magnetograph room at the hour considered and at the times of the absolute determinations of H in the month in question. The temperature coefficient of the H magnetograph is very small, and the correction comparatively unimportant.

The procedure in the case of V and I is less direct. Here the absolute observation gives I, while the curve gives V. The value of 1 centim. of ordinate is found as in the case of H by direct experiment, and the instrument is adjusted—usually once a year—so as to keep the sensitiveness as close as possible to 1 centim. $= 50\gamma$. Three or four absolute observations of I are made each month, usually on the days of the observations of H. The H observation is corrected to the time of the corresponding

I observation, by allowing for any change of curve ordinate and for any change of magnetograph room temperature that may have occurred meantime. Combining the observed values of I and of H, as corrected, one obtains an absolute value for V, and so can institute a comparison with the V curve and determine the value of its base line.

The hourly values of inclination are obtained by combining corresponding hourly values of H and V as given by the curves. The same inclinometer has been in use throughout the eleven years. The needles originally employed were damaged in 1899 and had to be replaced. The needles then introduced had been previously compared with the discarded pair, and no appreciable difference was detected.

§ 5. The methods of reduction just described had not been strictly adhered to in some of the earlier years. For instance, no satisfactory allowance had been made prior to 1894 for change in H between the hours of the H and I observations. This had the effect of depressing the values of V and I by something like 30γ and $1'$ respectively. Trouble also arose from an addition made to the Observatory building in 1892, introducing various discontinuities into the curves, which had not been altogether successfully dealt with at the time. Reference to the original records, however, enabled these defects to be removed fairly satisfactorily, and when this was done the more striking irregularities apparent in the secular changes of V and I between 1890 and 1894 mainly disappeared. There still remained rather a prominent discontinuity in the values for 1896. On examining into the matter, I found that in March, 1896, an old weak pair of bar magnets used for stroking the dip needles had been replaced by a much stronger pair. The secular change obtained by comparing the results for the six months immediately following the introduction of the new bar magnets with the results for the corresponding six months of the previous year amounted to only $1'$, whereas by comparing results from the six months immediately preceding the change with the results from the corresponding months of the previous year, and by comparing results from the six months immediately following the change with the results from the corresponding months of the subsequent year, one got values for the secular change which were in good agreement and averaged $2'\cdot 8$. This seemed to point pretty conclusively to a sudden discontinuity amounting to about $1'\cdot 8$, and simultaneous to all appearance with the introduction of the new bar magnets. For our present purpose it is immaterial whether the cause of this discontinuity was the introduction of the new bar magnets, or something else occurring approximately at the same time, so long as it is admitted that it is legitimate to apply a correction of the amount $1'\cdot 8$ indicated above. One's approval of any such correction must largely depend on one's estimate of the knowledge and judgment of the person who decides to apply it. It may be added, however, as evidence in its favour that it wiped out discontinuities in the 1896 values alike of V, I, and T. The consequence of the several corrections introduced was to alter the mean annual values of the elements as published in the annual 'Reports' to the following extent:—

Year	1890.	1891.	1892.	1893.	1894.	1895.	1896.
H	−4γ	—	+ 2γ	—	—	—	—
V	—	+72γ	+93γ	+96γ	+57γ	+57γ	+13γ
I	—	+2′·0	+2′·6	+2′·7	+1′·6	+1′·6	+0′·4

So far as can be judged, no correction is necessary for personal equation in the observations. During the whole period dealt with, the magnetic work has been under the charge of the Chief Assistant, Mr. T. W. BAKER, who has himself observed on the great majority of occasions. During 1891 to 1893 Mr. R. S. WHIPPLE observed pretty frequently, and since then, during Mr. BAKER's holidays, the horizontal force observations have been mainly taken by myself, and the declination and inclination observations by Mr. W. BOXALL. No certain difference has, however, manifested itself between the several observers.

§ 6. The mean annual values of the elements as corrected are given in Table II., and the corresponding secular changes appear in Table III. The values of N, W, and T were calculated from those of D, H, and I by the formulæ

$$N = H \cos D, \qquad W = H \sin D, \qquad T = H \sec I.$$

The action of the vertical-force magnetograph during the last two or three months of 1890 was not considered satisfactory at the time, and the corresponding results were discarded. Thus 1891 was the first year for which satisfactory mean annual values were obtainable for V and I. The mean values at the foot of Table II. are thus calculated for 10 years only in the case of V, T, and I, as against 11 years in the other cases. These mean values are wanted later, they do not possess any immediate significance in connection with secular change.

In Table III. I have added mean values of the secular changes at Parc St. Maur and Potsdam, for comparison with the mean changes at Kew. The Parc St. Maur data are from publications by MOUREAUX.* As data for 1900 were not available the means in this case are for the period 1890 to 1899 for D and H, and the period 1891 to 1899 for I and V. The Potsdam data are from a paper by ESCHENHAGEN;† they refer to exactly the same periods as the corresponding Kew data.

* 'Ann. du Bureau Central Météorologique de France,' 1897, "Mémoires," p. B. 65.

† 'Ann. der Physik,' vol. 6, 1901, p. 424.

TABLE II.—Mean Annual Values.

Year.	D.	I.	H.	W.	N.	V.	T.
	° ′	° ′					
1890	17 50·6	—	·18169	·05567	·17295	—	—
1891	17 41·9	67 33·2	·18193	·05531	·17332	·44034	·47648
1892	17 36·7	67 32·0	·18204	·05508	·17351	·44012	·47636
1893	17 28·8	67 29·0	·18238	·05478	·17396	·43992	·47625
1894	17 23·0	67 27·6	·18251	·05453	·17417	·43971	·47612
1895	17 16·8	67 25·4	·18278	·05429	·17453	·43958	·47608
1896	17 10·8	67 22·7	·18309	·05408	·17492	·43937	·47600
1897	17 6·4	67 19·6	·18342	·05395	·17532	·43906	·47583
1898	17 1·4	67 17·6	·18364	·05376	·17559	·43885	·47574
1899	16 57·1	67 14·7	·18393	·05363	·17594	·43852	·47553
1900	16 52·7	67 11·8	·18428	·05350	·17634	·43831	·47548
Means . . .	17 18·8	67 23·4	·18288	·05442	·17460	·43938	·47599

TABLE III.—Secular Changes.

Year.	D.	I.	H.	W.	N.	V.	T.
1890–1	′ −8·7	′ —	+24γ	−36γ	+37γ	—	—
1891–2	−5·2	−1·2	+11γ	−23γ	+19γ	−22γ	−12γ
1892–3	−7·9	−3·0	+34γ	−30γ	+45γ	−20γ	−11γ
1893–4	−5·8	−1·4	+13γ	−25γ	+21γ	−21γ	−13γ
1894–5	−6·2	−2·2	+27γ	−24γ	+36γ	−13γ	−4γ
1895–6	−6·0	−2·7	+31γ	−21γ	+39γ	−21γ	−8γ
1896–7	−4·4	−3·1	+33γ	−13γ	+40γ	−31γ	−17γ
1897–8	−5·0	−2·0	+22γ	−19γ	+27γ	−21γ	−9γ
1898–9	−4·3	−2·9	+29γ	−13γ	+35γ	−33γ	−21γ
1899–1900	−4·4	−2·9	+35γ	−13γ	+40γ	−21γ	−5γ
Mean annual change at—							
Kew	−5·79	−2·38	+25·9γ	−21·7γ	+33·9γ	−22·6γ	−11·1γ
Parc St. Maur . . .	−5·46	−1·60	+25·3γ	—	—	−0·3γ	—
Potsdam.	−5·23	−1·14	+22·7γ	—	—	+13·9γ	+21·8γ

§ 7. It will be observed that the mean secular changes in D and H at Parc St. Maur and Potsdam are very similar to those at Kew. There is even, in the case of these elements, a fair resemblance in the variations from year to year. All three stations, for instance, show a decidedly slackened rate of change in D since 1896. In 1898–9 and 1899-1900, the changes in D at Potsdam were absolutely identical with those at Kew; but in 1892–3 the change at Potsdam was less by 1′ than in either 1891–2 or 1893–4, whereas at Kew it was 2′ greater in 1892–3 than in the two adjacent years.

Kew and Potsdam agree in representing the change in H during 1891–2 as particularly small, and the changes in 1893–4 and 1897–8 were also at both places decidedly below the average. Again, we have specially large changes of H at Kew and Potsdam in 1892–3, 1895–6, 1896–7, and 1899–1900. Parc St. Maur agrees with Potsdam and Kew in having a specially small change of H in 1893–4, and specially large changes in 1892–3 and 1896–7; but it differs in showing an average change in 1891–2 and a less than average change in 1895–6.

In inclination and vertical force the mean data at the three stations are widely different; Parc St. Maur shows a nearly stationary value of V, as against a distinctly rising value at Potsdam, and a still more distinctly falling value at Kew. The inclination is diminishing at all three places, but the rate of fall appears considerably less at Potsdam than at Parc St. Maur, and considerably less at Parc St. Maur than at Kew.

Speaking generally, the secular changes in I, V, and T appear to have been much more irregular at Potsdam and Parc St. Maur than at Kew. Thus, at Parc St. Maur V rose 29γ in 1891–2 and fell 40γ in 1893–4; while at Potsdam V increased by 40γ in 1891–2, 43γ in 1892–3, and 74γ in 1899–1900, and fell 16γ in each of the three years 1893–4, 1894–5, and 1898–9. There are corresponding fluctuations, of course, in T. At Kew the accuracy of the estimate of secular change in V and T is really dependent in the main on the accuracy of the determination of I. An error of $1'$ in I—and such an error might arise from very slight deterioration of either circle or needles—means an error of over 30γ in both V and T. Taking this into account, I do not think much weight can be assigned to the values of the secular change for individual years. The comparative smoothness of the V and T data in Table III. is probably not more a tribute to the uniformity of Nature than to the care and skill of Mr. BAKER as an observer.

The theory has been advanced that secular changes are particularly large in years of sun-spot maximum. The data in Table III. can hardly be said to support this. The changes of D were certainly larger in 1892–6 when sun-spots were numerous, than in 1898–1900 when they were few; but the largest change in D occurred in 1890–91 near sun-spot minimum, and in 1893–4 about sun-spot maximum the secular change was about average in D and particularly small in H and I.

§ 8. To assist in the comprehension of the secular change I have drawn a curve, fig. 1, which shows the change from year to year in the direction of an imaginary magnet, freely suspended from its C.G. so as to point along the line of total force. The curve may be supposed drawn by a style attached to the dipping end of the magnet, the paper being supposed to lie on a plane tangential to a sphere, concentric with the magnet, at the point answering to the mean position of the dipping end during the period 1891 to 1900. With increasing I, the dipping end would move towards an observer who stood with his back to the south, looking at the paper. The west, *i.e.*, the direction of + D, would then be on the observer's left. The crosses mark the positions of the end of the magnet at the middle of the years specified. The magnet or style is supposed to be so long that the difference between arc and tangent may be neglected, so that equal lengths on the paper represent equal changes of direction in the magnet. The curve, it will be noticed, does not depart very far from a straight line.

§ 9. The above graphical method throws no direct light on the absolute magnitudes of the changes of the components of force. It is, however, of some interest to know

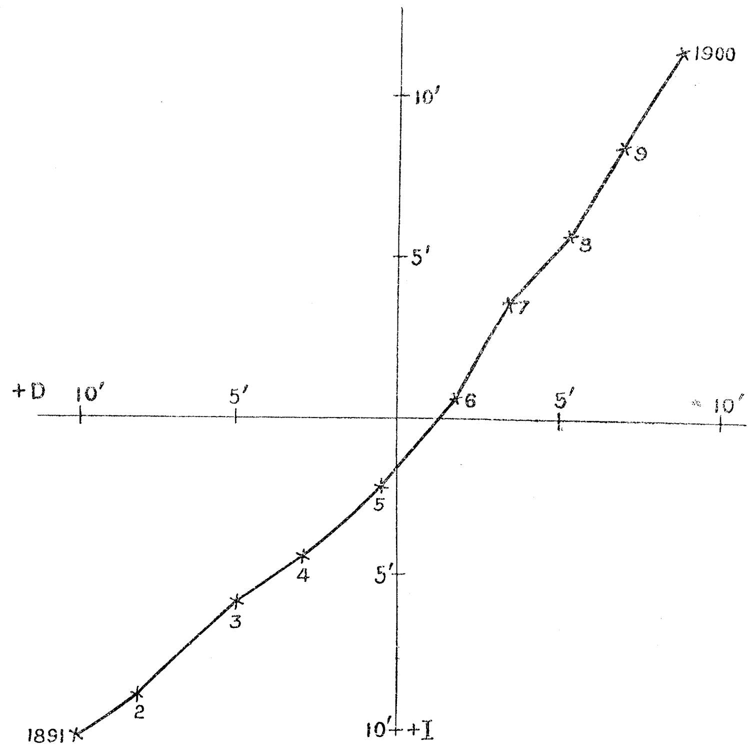


Fig. 1.

the direction and amplitude of the disturbing force to which the secular change may be attributed. To find this we may proceed as follows :—

Let N, W, V represent the values of the northerly, westerly, and vertical components of force at the beginning of a period, at the end of which the values have become $N + \delta N$, $W + \delta W$, $V + \delta V$. The total force meantime has changed from T to $T + \delta T$.

δN, δW, δV represent the components of a force δR (totally different usually from δT) which may be regarded as producing the secular change. If secular change data for individual years were sufficiently reliable, one would naturally take the δR answering to a single year's change as measuring in direction and magnitude the force to which the secular change may be attributed.

In general, however, it will be best to calculate a δR for a considerable number of years interval, and to regard its line of action as indicating the general direction of the secular change force throughout the period.

In the present instance we get for the period 1891 to 1900, at Kew,

$$\delta N = +302\gamma, \quad \delta W = -181\gamma, \quad \delta V = -203\gamma.$$

Thence we find $\delta R = 406\gamma$, and its line of action makes with three rectangular axes drawn respectively to the north, the *east*, and vertically *upwards* the angles 42°·0, 63°·5, and 60°·0.

The projection of the line of action of δR on the horizontal plane is inclined to the geographical meridian at the angle 30°·9, running in a north-*easterly* direction.

Annual Inequality.

§ 10. If the secular change proceeded at a uniform rate throughout the year, the mean value E_n of any element answering to the middle of the n^{th} month of the year should be derivable from the mean value $\overline{E}$ for the whole year by the formula

$$E_n = \overline{E} + (2n - 13)\, s/24,$$

where s is the secular change for the entire year.

This neglects the differences between the lengths of months, as being for the present purpose immaterial.

Conversely, if one applies to $E_n - \overline{E}$ the correction $-(2n - 13)\, s/24$, one eliminates the effect of a regularly progressive secular change, and obtains what is known as the "Annual Inequality."

In practice, complications arise from the apparent variability in the secular change from year to year. To illustrate the consequences of such irregularity, take the simple hypothetical case where the secular change of declination proceeds uniformly from January 1 to December 31 of a year at the rate of 12′ a year, and then proceeds for the next twelve months at the uniform rate of 6′ a year. From the mean values for the two years we should deduce a secular change, not of 12′ or of 6′, but of 9′ a year, and if we corrected the two years' monthly values independently, on the assumption of a secular change of 9′ a year, we should deduce for each year a wholly fictitious annual inequality. Combining the monthly values for the two years we should in this case conclude, rightly enough, that there was no true annual inequality, but that is merely an accident of the particular hypothesis. The illustration will show how uncertain is the physical interpretation to be put on an apparent annual inequality in the case of an element whose secular change is irregular, unless we deal with mean monthly values from a large number of years.

The results in Table IV. have been obtained by combining 10 years' results for I, V, and T, and 11 years' results for the other elements. The data assigned to the "middle of the month" are the values actually obtained for the differences between the mean monthly values and the mean annual value after the application of the corrective term $-(2n - 13)\,s/24$. The value assigned to s in each case was that given in Table III. as the mean for the 10 or 11-year period considered. The results under the heading "beginning of month" are each the arithmetic mean of two adjacent mid-month values; for example, the value assigned to January refers to January 1, and is the mean of the values for mid-December and mid-January. The mid-month data are the most suitable for use in a critical examination into the true nature of the phenomena; but, assuming the existence of a true annual inequality, the beginning of the month data are probably the more reliable measures of its amplitude.

In all the force components, unity represents 1γ.

TABLE IV.—Annual Inequality.

	Middle of month.				Beginning of month.						
	D.	I.	H.	V.	D.	I.	H.	W.	N.	V.	T.
January . .	+0′·08	−0′·03	−0·9	− 0·4	+0′·17	−0′·16	−0·1	+0·8	−0·4	− 4·1	−3·8
February . .	+0·48	−0·07	+1·9	+ 2·1	+0·28	−0·05	+0·5	+1·6	0·0	+ 0·8	+1·0
March . . .	+0·03	+0·53	−0·1	+14·4	+0·25	+0·23	+0·9	+1·6	+0·4	+ 8·2	+8·0
April . . .	−0·31	+0·18	−0·6	+ 5·7	−0·14	+0·35	−0·3	−0·8	−0·1	+10·0	+9·1
May . . .	−0·39	−0·15	+2·7	+ 1·5	−0·35	+0·02	+1·0	−1·5	+1·6	+ 3·6	+3·7
June . . .	−0·47	−0·35	+5·2	− 0·3	−0·43	−0·25	+4·0	−1·0	+4·5	+ 0·6	+2·1
July . . .	−0·30	−0·13	+3·8	+ 5·4	−0·38	−0·24	+4·5	−0·6	+4·9	+ 2·5	+4·0
August . .	+0·08	−0·19	+0·5	− 3·5	−0·11	−0·16	+2·2	+0·1	+2·2	+ 0·9	+1·7
September .	+0·29	+0·20	−4·4	− 6·6	+0·19	+0·01	−2·0	+0·4	−2·2	− 5·1	−5·4
October . .	+0·06	0·00	−4·4	− 7·6	+0·18	+0·10	−4·4	−0·4	−4·5	− 7·1	−8·3
November .	+0·17	+0·18	−4·4	− 2·7	+0·12	+0·09	−4·4	−0·7	−4·4	− 5·2	−6·5
December. .	+0·26	−0·29	+0·6	− 7·8	+0·22	−0·05	−1·9	+0·5	−2·1	− 5·2	−5·6

§ 11. If we take the beginning of the month data in Table IV. we have the following results:—

	D.	I.	H.	W.	N.	V.	T.
Range of annual inequality . .	0′·71	0′·60	8·9γ	3·1γ	9·4γ	17·1γ	17·4γ
Range ÷ secular change . . .	0·12	0·25	0·34	0·14	0·28	0·76	1·57

The ratio borne by the range of the annual inequality to the mean secular change is certainly largest in those elements in which the uncertainties are greatest.

In the case of D and H (or of W and N) we are dealing with elements whose secular change has been comparatively regular, and there are other grounds for regarding the annual inequalities obtained for them as the most reliable.

They show a pretty close resemblance in general character to results obtained previously* for Kew from the five years 1890–4. At the same time, the ranges now found are somewhat conspicuously less than the ranges, 1′·22 in D and 12·9γ in H, obtained on the previous occasion.

In the case of D and H, the variation in the mid-monthly values in Table IV. seems altogether too regular to be ascribed to chance, and even in I and V something more than chance seems involved. It must be allowed, however, that in these last two elements the value for March stands out in a way one can hardly suppose to be truly representative of Nature, and in the case of V, the tendency to negative values in the latter half of the year, and the apparent jump from December to January, are somewhat suggestive of an under-estimate of the true secular change.

§ 12. One possibility to which attention should be drawn is that an annual inequality may be in no way ascribable to errors of observation, or of estimation of the secular change, and yet be absolutely fictitious so far as terrestrial magnetism is concerned. For example, if the distant mark used in absolute observations of D shifted in any way dependent on the temperature of the air, or on the temperature or moisture of the soil, an apparent annual inequality would ensue. A similar result would happen in the case of H if the law of variation with temperature in the moment of inertia of the collimator magnet were wrongly assumed. In the present case I have applied an elaborate check—whose description would take me too far afield—calculated to disclose any appreciable uncorrected effect of temperature on absolute observations of H, and the conclusion it led to was that if any such effect existed it was small compared to the range found for the annual inequality. It is also noteworthy that in all the elements the beginning of the month data indicate a large semi-annual term in the annual inequality, which is not what one would expect to find if the true cause of the phenomena were moisture or temperature.

Non-cyclic Effect.

§ 13. In virtue of the secular changes shown in Table III., the value of an element should exceed its value 24 hours earlier, on the average throughout the period now dealt with, by the following amounts :—

D.	I.	H.	W.	N.	V.	T.
−0′·016	−0′·007	+0·07γ	−0·06γ	+0·09γ	−0·06γ	−0·03γ

These quantities are all much less than can be directly measured on the magnetograms. Such as they are, however, they contribute to what I have termed the "non-cyclic effect."†

* 'British Association Report for 1895,' p. 226.

† 'British Association Report for 1895,' p. 210.

Again, the annual inequality, when such exists, implies an increment or decrement in the course of 24 hours, varying with the time of year; and if Table IV. may be trusted, this increment at certain seasons should be considerably larger than that due to the secular change. But even the most prominent monthly change in the table, viz., from $2{\cdot}1\gamma$ to $14{\cdot}4\gamma$ in the case of V, would represent an increment of only about $0{\cdot}4\gamma$ in a day. Any increment, of course, due to annual inequality would change in sign with the season, and would cut out for the year as a whole.

There are other known causes tending to introduce a difference between the values of the magnetogram ordinates answering to the beginning and end of a day. There are the irregular movements visible in most curves, especially at times of sun-spot maximum, and imperfections in the temperature corrections applied, at least in the case of V. These two last mentioned sources would, however, only introduce irregularities into the monthly values of the non-cyclic effect, without appreciably affecting the mean for the year.

A more troublesome source of non-cyclic effect is instrumental change. In the case of D there does not seem to be any appreciable uncertainty on this head. This might be inferred from the nature of the declination magnetograph, but it is at least desirable to have observational confirmation. During the eleven years dealt with, the magnet mirror had been re-adjusted only once, viz., in 1898, when its azimuth was altered so as to increase the curve ordinate by the equivalent of about 36′ of declination. Allowing for this, the curve ordinate had diminished by the equivalent of about 53′, between the taking of absolute observations in January, 1890, and December, 1900, which showed an actual decrease of 63′ in D. This leaves only about 10′ of shift unaccounted for by true secular change. Spread over 11 years, this would represent an exceedingly small non-cyclic effect. In the case of the bifilar magnetograph also, only one slight artificial alteration had been made in the distance between the trace and the base line. Allowing for this, the curve ordinate increased by about 65γ, whilst H really increased by about 280γ. The deficiency 215γ represents presumably in the main decrease in the moment of the bifilar magnet. Spread over 11 years, this represents a mean non-cyclic effect of about $0{\cdot}05\gamma$ per day. This, as we shall see, is only about a sixtieth of the average non-cyclic effect actually observed on quiet days. If we combine the effects due to true secular change and instrumental causes, we have a mean non-cyclic effect of only about $0{\cdot}02\gamma$ per day. This is in the same direction as the non-cyclic effect actually observed in H, but is wholly insignificant.

Results confirmatory of our conclusion as to the smallness of the non-cyclic effect in H, due to secular change and instrumental causes combined, were obtained from measurements of the curve ordinates at the beginning and end of a series of individual years.

In the case of V, the magnet mirror had been altered seven or eight times in the course of 11 years, usually early in January, at the time of the scale determinations.

As the balance had usually been altered at the same time, it was difficult to arrive at direct conclusions as to the true effect on the curve ordinates. A comparison, however, of the curve ordinates after the scale determinations in January with those in the following December, on seven years when no intermediate change had been made, showed a decrease in the ordinate due to instrumental causes at the average rate of 24γ a year. The change was in the same direction each year. An annual change of -24γ gives a mean non-cyclic effect of $-0{\cdot}07\gamma$ per day. Combining instrumental causes with true secular change, we get a non-cyclic effect of about $-0{\cdot}13\gamma$ per day. This represents, as we shall see, about 15 per cent. of the average non-cyclic effect actually observed on quiet days.

§ 14. There is unavoidably some uncertainty in the estimates of artificial non-cyclic effects, and I have attempted no correction to the results as observed.

The H and V magnets would probably be more exposed to losses of magnetism during magnetic storms than on ordinary occasions, and the chances are that any estimate we should make of the contribution from instrumental causes to the observed non-cyclic effect on quiet days would be in excess, in the case at least of V. Again, as will be seen presently, we have to correct the observed readings of the curves for non-cyclic effect, and, so far as the corrections are concerned, what we have to deal with is the entire effect, whatever its sources. It is desirable to have on record the actual corrections made, so that anyone can reproduce the actual observation results who wishes to do so.

The quantities then actually recorded under the heading "non-cyclic effect" in the following tables V. to X. represent the algebraic excess of the recorded value of an element at the second midnight over that at the first midnight of a "quiet" day, irrespective of the source to which the increment is due.

In Table X., as subsequently, the year is sub-divided into three seasons—

"Winter," comprising the four months November, December, January, February;
"Equinox" ,, ,, ,, March, April, September, October;
"Summer" ,, ,, ,, May, June, July, August.

TABLE V.—Non-Cyclic Effect in D.

	1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.	1898.	1899.	1900.	Monthly means.	Number of months when effect—		
													+	0	−
January . .	+0·1	+0·1	+0·2	+2·5	+0·3	+0·6	−0·8	+0·1	0·0	+0·1	−0·2	+0·27	8	1	2
February .	−0·8	+0·5	+0·2	−0·1	+2·1	+0·1	−0·6	−0·2	−0·4	−0·2	+0·2	+0·07	5	0	6
March . .	−0·1	+0·6	+0·9	−0·2	+0·1	−0·2	0·0	+0·2	+0·3	+0·2	0·0	+0·16	6	2	3
April . . .	−0·3	+0·6	+0·1	0·0	+0·5	−0·2	+0·6	−0·1	+0·2	+0·2	−0·3	+0·12	6	1	4
May . . .	−0·3	+1·0	−0·1	+0·5	+0·2	−0·9	−0·3	−0·5	−0·4	0·0	+0·2	−0·055	4	1	6
June . . .	−0·7	+0·1	+0·1	−0·2	−0·6	+0·3	+0·8	−0·4	−0·1	+0·1	+0·1	−0·045	6	0	5
July . . .	−0·5	+0·2	−0·6	+0·4	−1·7	+0·8	+0·1	0·0	0·0	+0·1	−0·2	−0·13	5	2	4
August . .	−0·4	+0·1	−0·7	0·0	−0·5	−0·3	+0·2	+0·4	+0·4	+0·1	+0·1	−0·055	6	1	4
September .	−0·2	+0·4	+0·2	−0·4	+0·8	−0·1	−0·4	+0·4	−0·3	0·0	−0·1	+0·03	4	1	6
October . .	−0·5	+0·4	+1·2	−0·4	0·0	−0·5	−0·1	−0·3	−0·1	−0·1	0·0	−0·04	2	2	7
November .	0·0	−0·7	+0·2	+1·1	+0·7	−0·2	−0·1	−0·2	+0·5	+0·2	+0·4	+0·17	6	1	4
December .	−0·6	+0·2	0·0	−0·1	−0·1	0·0	+0·5	+0·1	+0·2	−0·1	+0·1	+0·02	5	2	4
Mean for year .	−0·36	+0·29	+0·14	+0·26	+0·15	−0·05	−0·01	−0·04	+0·025	+0·05	+0·025	+0·044	—	—	—

TABLE VI.—Non-Cyclic Effect in I.

	1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.	1898.	1899.	1900.	Monthly means.	Number of months when effect—		
													+	0	–
January . .	+0′·1	−0′·4	−0′·6	−1′·0	−0′·4	−0′·5	0′·0	0′·0	−0′·1	−0′·3	−0′·2	−0′·31	1	2	8
February .	−0·2	+0·1	−0·4	−0·3	−0·8	−0·5	−0·3	−0·1	−0·1	−0·3	−0·3	−0·29	1	0	10
March . .	+0·4	−0·1	−0·7	0·0	−0·1	−0·5	−0·6	−0·1	−0·7	−0·1	−0·4	−0·26	1	1	9
April. . .	−0·2	−0·3	−0·5	0·0	+0·1	−0·3	−0·8	−0·8	−0·1	−0·3	−0·3	−0·32	1	1	9
May . . .	−0·6	−0·3	−0·2	−0·8	+0·2	+0·1	−0·8	−0·5	0·0	−0·2	−0·5	−0·33	2	1	8
June . . .	+0·1	−0·2	−0·1	−0·6	0·0	−0·5	−0·3	−0·2	0·0	−0·1	+0·2	−0·155	2	2	7
July . . .	0·0	−0·1	−0·5	0·0	−0·2	−0·5	−0·2	−0·2	−0·3	−0·3	+0·2	−0·19	1	2	8
August . .	−0·2	−0·2	−0·7	−0·4	−0·1	0·0	+0·1	−0·3	−0·2	−0·4	−0·3	−0·245	1	1	9
September .	−0·1	−0·3	−0·2	−0·1	+0·1	−0·5	−0·2	−0·3	−0·6	−0·1	0·0	−0·21	1	1	9
October . .	−0·3	−0·2	−0·5	−0·3	−0·5	+0·1	0·0	−0·3	−0·2	−0·2	−0·1	−0·23	1	1	9
November .	—	−0·1	−0·5	−0·5	−0·4	−0·5	−0·1	−0·2	−0·2	−0·3	0·0	−0·28	0	1	9
December .	—	−0·1	0·0	−0·2	0·0	−0·4	0·0	−0·3	−0·2	−0·2	+0·1	−0·13	1	3	6
Mean for year .	−0·10	−0·18	−0·41	−0·35	−0·175	−0·33	−0·27	−0·275	−0·225	−0·23	−0·13	−0·245	—	—	—

TABLE VII.—Non-Cyclic Effect in H. (Unity ≡ 1γ.)

	1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.	1898.	1899.	1900.	Monthly means.	Number of months when effect—		
													+	0	−
January	−2	+3	+7	+13	+4	+5	+2	+1	+3	+3	+2	+3·7	10	0	1
February	+6	0	+6	+3	+12	+7	+5	+2	+1	+4	+3	+4·5	10	1	0
March	+1	+4	+6	0	+1	+5	+6	+3	+7	+1	+5	+3·5	10	1	0
April	+5	0	+3	+2	−2	+4	+10	+10	+1	+5	+4	+3·8	9	1	1
May	+6	+6	0	+8	+2	+1	+7	+5	+2	+1	+6	+4·0	10	1	0
June	−1	+1	+2	+5	0	+6	+4	+3	−1	+1	−2	+1·6	7	1	3
July	0	0	+6	+1	+3	+6	+3	+2	+5	+5	−2	+2·6	8	2	1
August	+4	+4	+10	+3	+2	−1	+1	+3	+2	+5	+2	+3·2	10	0	1
September	+1	+4	+4	+5	+3	+6	+5	+4	+7	+1	0	+3·6	10	1	0
October	+2	+5	+9	+2	+7	+5	0	+3	+2	+3	+2	+3·6	10	1	0
November	+5	+2	+7	+5	+7	+6	0	+3	+3	+5	0	+3·9	9	2	0
December	+1	−1	+4	+1	+1	+3	+2	+5	+2	+4	−1	+1·9	9	0	2
Mean for year	+2·3	+2·3	+5·3	+4·0	+3·3	+4·4	+3·8	+3·7	+2·8	+3·2	+1·6	+3·34	—	—	—

TABLE VIII.—Non-Cyclic Effect in V. (Unity $\equiv 1\gamma$.)

	1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.	1898.	1899.	1900.	Monthly means.	Number of months when effect— +	0	−
January	− 4	− 8	− 5	− 3	− 5	− 6	+ 2	+ 1	0	− 2	− 2	− 2·9	2	1	8
February	+ 5	+ 3	+ 1	− 1	− 1	− 4	+ 2	− 1	0	− 2	− 2	0·0	4	1	6
March	+ 18	+ 7	− 13	0	0	− 4	− 8	+ 1	− 5	− 2	− 1	− 0·6	3	2	6
April	+ 8	− 10	− 8	+ 4	− 4	− 3	− 7	− 3	− 1	− 1	− 4	− 2·6	2	0	9
May	− 7	+ 1	− 9	− 7	+ 10	+ 5	− 14	− 3	+ 2	− 3	− 3	− 2·5	4	0	7
June	− 1	− 4	+ 1	− 7	− 2	− 4	− 3	+ 2	− 1	− 2	+ 3	− 1·6	3	0	8
July	− 2	− 2	− 1	+ 2	− 2	− 3	0	0	+ 1	− 1	+ 3	− 0·5	3	2	6
August	+ 5	+ 1	− 2	− 8	+ 2	− 3	+ 5	− 4	− 1	− 2	− 3	− 0·9	4	0	7
September	0	− 3	+ 1	+ 7	+ 14	− 1	+ 2	0	− 4	− 2	+ 1	+ 1·4	5	2	4
October	− 7	+ 3	+ 2	− 6	− 3	+ 14	+ 2	− 3	− 1	− 1	+ 1	+ 0·1	5	0	6
November	—	+ 2	− 1	− 7	+ 4	− 4	− 2	+ 1	+ 1	+ 2	0	− 0·4	5	1	4
December	—	− 5	+ 10	− 5	+ 2	− 5	+ 4	+ 1	+ 1	+ 2	+ 1	+ 0·6	7	0	3
Mean for year	+ 1·5	− 1·25	− 2·0	− 2·6	+ 1·25	− 1·5	− 1·4	− 0·7	− 0·7	− 1·2	− 0·5	− 0·84	—	—	—

TABLE IX.—Non-cyclic Effect.

Year.	Declination. Number of months.			Inclination. Number of months.			Horizontal force. Number of months.			Vertical force. Number of months.		
	+	0	–	+	0	–	+	0	–	+	0	–
1890 . . .	1	1	10	3	1	6	9	1	2	4	1	5
1891 . . .	11	0	1	1	0	11	8	3	1	6	0	6
1892 . . .	8	1	3	0	1	11	11	1	0	5	0	7
1893 . . .	4	2	6	0	3	9	11	1	0	3	1	8
1894 . . .	7	1	4	3	2	7	10	1	1	5	1	6
1895 . . .	4	1	7	2	1	9	11	0	1	2	0	10
1896 . . .	5	1	6	1	3	8	10	2	0	6	1	5
1897 . . .	5	1	6	0	1	11	12	0	0	5	2	5
1898 . . .	5	2	5	0	2	10	11	0	1	4	2	6
1899 . . .	7	2	3	0	0	12	12	0	0	2	0	10
1900 . . .	6	2	4	3	2	7	7	2	3	5	1	6
Totals. .	63	14	55	13	16	101	112	11	9	47	9	74

TABLE X.—Non-cyclic Effect, Seasonal Values (Means from 11 Years). Unit = 1γ in case of Force.

	D.	W.	H.	N.	I.	V.	T.
Winter. . . .	+0′·134	+1·72	+3·50	+3·13	−0′·252	−0·68	+0·72
Equinox . . .	+0·068	+1·44	+3·66	+3·39	−0·254	−0·46	+0·99
Summer . . .	−0·070	+0·50	+2·86	+2·84	−0·230	−1·39	−0·18
Year	+0·044	+1·22	+3·34	+3·12	−0·245	−0·84	+0·51

§ 15. The declination (see Table V.) is the element in which the non-cyclic effect is smallest and most variable in sign. The D curves are read only to the nearest 0′·1, so that apparent non-cyclic changes of this amount *on individual days* possess little, if any, significance. If we take the last six years in Table V., we see that the mean non-cyclic effect for the year in no case exceeds 0′·05, and if we combine these six years, we obtain a mean non-cyclic effect which absolutely vanishes. Again, while the mean non-cyclic effects for 1890 and 1891 are of substantial size, they are opposite in sign.

These facts are certainly calculated to rouse suspicions as to the *bonâ fide* nature of the non-cyclic effect in D. There are, however, a variety of considerations which point to its having a real physical origin.

In the first place, the contribution of the true secular change to the mean non-cyclic effect in D, viz., − 0′·016, is in the opposite direction to the resultant effect, so that the daily change which we have to account for really amounts on the average of the eleven years to $0'\cdot044 + 0'\cdot016$ or $0'\cdot06$. Again, the annual inequality is in general such as to diminish the numerical values found for the non-cyclic effect. In fact we see from Tables IV. and V. that the non-cyclic effect is positive in March and April, the months in which the annual inequality is changing fastest from positive to negative, while it is negative in July and August, months in which the annual inequality is changing from negative to positive.

This brings us to another suggestive feature. Four of the five months in which the mean non-cyclic effect is negative are summer months, while it is positive in six successive months, November to April, including all the mid-winter months. Then, as regards differences of sign in different years, sun-spots were exceptionally few in 1890, and though fairly numerous were rapidly diminishing in 1895, 1896, and 1897, the other three years in which the mean non-cyclic effect was negative. On the other hand, 1891 to 1894, the years in which the non-cyclic effect has its largest positive values, were years in which sun-spots were either very numerous or increasing rapidly in number. Thus I think that whilst it would be unwise to dogmatise as to the cause of the apparent non-cyclic effect in D, the evidence distinctly points to the conclusion that it is a true magnetic phenomenon which varies, however, with the season of the year, and with the position of the year in the sun-spot cycle.

§ 16. In the case of I (see Table VI.), the non-cyclic effect is evidently no chance phenomenon. The means for every year of the eleven, and for every month of the twelve, alike come out negative, and the preponderance of individual months in which the effect is negative is overwhelming. The true secular change is here in the same direction as the apparent non-cyclic effect, and so tends to increase it, but only to an insignificant extent. In fact it requires only 10 average quiet days to bring about a decrease in I equal to the secular change observed in an entire year.

The absolute magnitude of the non-cyclic change in I, as we see from Table X., does not vary very conspicuously with the season of the year, but relative to the range of the diurnal inequality it is most important in winter. In January, in fact, it averaged fully 30 per cent. of the range of the proper cyclic diurnal variation.

§ 17. The non-cyclic effect in H, see Table VII., is as prominent as in I. Here also it is increased by the contribution from the secular change, but 8 average quiet days suffice to produce an increment in the force equal to the full secular change for the year. The non-cyclic effect in H is on the whole wonderfully uniform throughout the year, but it appears somewhat reduced at midsummer. Relative to the range of the diurnal inequality, it is most important in winter; in January, in fact, it averaged over 20 per cent. of the observed range.

In V, see Table VIII., the non-cyclic effect is considerably less conspicuous than in H or I, but more conspicuous than in D. There is a decided preponderance of months in which it has the negative sign, and the mean annual value is positive in only 2 out of the 11 years. According to Table X., the magnitude of the non-cyclic effect in V is considerably dependent on the season of the year, being greater in summer than in winter. We have already seen that an appreciable though small fraction of the effect in V is ascribable to secular change and instrumental causes: and if these causes are as potent in winter as in summer, the difference between the phenomena observed in summer and winter is all the more noteworthy.

In the case of W, N and T, I have confined myself to mean seasonal values (see Table X.). The equivalent in force of 1′ in D is 5·3 γ approximately. Hence it will be seen that the mean non-cyclic effect in W, though much less than in H, is larger than in D. The non-cyclic effect in N approaches closely that in H.

In T the non-cyclic effect appears to be but little larger than in D, and it also shows the peculiarity of being positive in winter but negative in summer. The secular change and the instrumental causes described in dealing with V would tend to produce a slight (algebraic) reduction in the results obtained for T.

Solar Diurnal Inequality.

§ 18. By the diurnal inequality is meant the *periodic* change taking place in the value of an element throughout the 24 hours. In the case of a magnetic element on quiet days, there is as we have seen an appreciable non-cyclic effect, in virtue of which the value recorded at the second midnight of a day exceeds that recorded at the first midnight by a quantity, say N, which may be positive or negative. To obtain the proper *periodic* change, N must be eliminated, for unless this is done a diurnal inequality is obtained which is not periodic, which presents a discontinuity between two successive hourly values, and which is partly dependent on which hour of the 24 we select as the first hour of our day. If we assume N to result from some cause operating uniformly throughout the day, and count our day from midnight, 0 hours, to midnight, 24 hours, the necessary elimination is effected by applying to each hourly value the correction

$$N\,(12 - n)/24,$$

where n is the hour counted from midnight (0 hours).

This correction brings of course the values answering to hours 0 and 24 into harmony, and leaves unaltered the mean value M of the element for the day as given by

$$M = \tfrac{1}{24}\left\{\tfrac{1}{2}[0] + \tfrac{1}{2}[24] + [1] + \ldots + [23]\right\},$$

where $[n]$ represents the value of the element at hour n. When dealing with hourly values from a single day, or as in the present case with hourly means from 5 days a month, we really do not know how the non-cyclic effect comes in, and our method of elimination *may* introduce a factor not present in diurnal inequalities based on all days of the month.

Another source of slight uncertainty may be noticed. The magnetic elements have in all probability a lunar as well as a solar diurnal variation. The incidence of the selected 5 quiet days in the lunar month is largely a matter of accident, and consequently the way in which the lunar variation affects hourly values based on the selected 5 quiet days must vary from month to month. The lunar influence appears however to be so small that any uncertainty on this ground must be trifling, even in the case of data from a single year, and when we are dealing with data from 5 or 6 years, the uncertainty should be no greater than in the case of data derived from all days in a single month.

A further point that is more fully dealt with in § 31 need only be mentioned now. The hours to which the curve measurements refer are G.M.T., whereas a true solar diurnal variation is connected presumably with the true local solar time. The fact that noon at Kew is $1\frac{1}{4}$ minutes later than noon at Greenwich is comparatively unimportant, as the difference is small and affects all seasons of the year alike, but the considerable annual range—some 31 minutes—in the "equation of time" must be borne in mind.

§ 19. A really serious difficulty remains to be mentioned. The Kew magnetographs are placed in a room partly underground in the basement of the Observatory, where the annual variation of temperature, though much less of course than in an unprotected upstairs room, still amounts to some 20° F. At some seasons of the year this implies a somewhat troublesome change of temperature from day to day, but speaking generally there is seldom a difference of more than 1° F., and usually only a difference of a few tenths of a degree between the mean temperature on successive days. It was thus possible to allow pretty satisfactorily for the difference between the mean temperature of the room on the five quiet days a month and on the days of the absolute observations, from eye readings of a mercury thermometer placed under the glass shade containing the vertical-force magnet. Up to the end of 1896 the readings of this thermometer were the only direct source of information as to the temperature of the magnetographs. During 1896 the thermometer had been read usually thrice a day, at 10 A.M., 4 P.M., and 10 P.M. Prior to that it had been read twice a day, but in the earlier years of the period only once a day. In the end of 1896 the late Kew Committee, on my initiative, introduced a thermograph. It was so situated and protected as to give readings in close accordance with those of the mercury thermometer—still read thrice daily—and temperature corrections based on its readings have been applied from 1897 onwards to the H and V curves. In the case of the H magnetograph the temperature coefficient is little over 1γ for 1° F., and the temperature correction is very small. In the case of V the correction is distinctly appreciable. An examination however of the thermograph results obtained early in 1897—at the season, as it has turned out, when the diurnal variation of temperature is least—and a comparison of the annual and semi-annual magnetic inequalities for 1896 and 1897, led me unfortunately to the conclusion that the neglect of the diurnal variation of temperature in previous years was less serious than I had feared. Influenced by this investigation, I accepted the data for 1890 to 1896 as published in the annual 'Reports,' and all the tables of diurnal inequalities, and numerous others, were originally calculated on this basis. The results so obtained were utilised in the "Preliminary Note on the Relationships between Sun-spots and Terrestrial Magnetism," published in the 'Roy. Soc. Proc.,' vol. 71, p. 221. Subsequently I noticed that the ranges in the mean annual inequalities for V for 1898 and 1899 appeared larger relative to the ranges in earlier years than one would have anticipated from the relationships I had observed between Sun-spots and Terrestrial Magnetism, and I had little difficulty in tracing the cause to the neglect of the temperature correction in the earlier years. Owing to the smallness of the temperature coefficient in the H magnetograph, the absence of a temperature correction in the data for years prior to 1897 is of little importance. It became obvious, however, that the earlier years' data for V must be corrected or dispensed with. The correction has entailed a great amount of labour, involving the complete recalculation of all the diurnal inequalities for V and I and of the corresponding Fourier coefficients, &c. The thermograph records from 1897 to 1901 gave smooth diurnal inequalities of temperature for each month of the year. The range varied largely with the season, being much less at midwinter than at midsummer, but the hours of maximum and minimum and the general features were nearly independent of the time of year. The magnetograph room and its environment remained unchanged from 1890 to 1901, and the readings from the mercury thermometer throughout the period confirmed the view that if the mean diurnal inequalities of temperature obtained for the several months of the year since 1897 were assumed to apply to years prior to 1897, satisfactory average corrections would be obtained. This accordingly has been done, and I do not think that the results now published are affected by any serious source of uncertainty.

§ 20. Diurnal inequalities for D, I, H, V, N, W, and T appear in Tables XI. to XVII. In the case of the first four elements inequalities are given for each month of the year. The values ascribed to any one month, say January, were obtained by adding together the hourly values from each January of the period included, applying a non-cyclic correction to the sums thus formed, and then taking the arithmetic means. The annual inequality for the year was obtained by summing up the mean hourly values for the 12 months—as corrected for non-cyclic effect—and taking their means. In the case of D and H mean inequalities are given, not merely for the whole 11 years, but for two combinations of years, viz., 1890, 1899, and 1900, years of few sun-spots, and 1892 to 1895, years of great sun-spot frequency.

The inequalities for N, W, and T were derived by means of the formulæ :—

$$\delta N = \cos D\, \delta H - H \sin D\, \delta D,$$
$$\delta W = \sin D\, \delta H + H \cos D\, \delta D,$$
$$\delta T = \cos I\, \delta H + \sin I\, \delta V,$$

where δN, for instance, represents the hourly value in the diurnal inequality of N, which corresponds to the values δH and δD, answering to the same hour, in the inequalities of H and D. To the necessary degree of accuracy, we may ascribe to D, H, and I in these formulæ their mean values for the period considered, and the formulæ actually used were (see Table II.) :—

$$\delta N = {\cdot}955\,\delta H - 1{\cdot}58\,\delta D,$$
$$\delta W = {\cdot}298\,\delta H + 5{\cdot}08\,\delta D,$$
$$\delta T = {\cdot}385\,\delta H + {\cdot}923\,\delta V.$$

Here δD is supposed to be measured in minutes of arc. In the case of N, W, and T, inequalities are given for only three months of the year, selected one from each of the three seasons; in the case of N and W mean inequalities are given for two groups of years, as well as for the whole 11 years.

In a good many cases there are two distinct maxima and two distinct minima in the inequalities. All maxima and minima whose existence seems absolutely unmistakable are indicated by heavy type, but in cases where the second maximum and minimum are missing or doubtfully represented only the extreme values for the day are so marked. In the case of D the terms "maximum" and "minimum" are applied respectively to an extreme westerly and an extreme easterly position of the magnet. This is of course purely arbitrary, and is opposed to the practice of a good many writers, who measure declination positively to the east. As a mathematician, I am perfectly alive to the weight of the arguments advanced in support of this practice, which has its conveniences when mathematical calculations are being made involving the use of Cartesian co-ordinates. At a station, however, such as Kew, where the declination is westerly, and the prominent feature is the rapid forenoon movement of the needle to the west, it is much more convenient for descriptive purposes to take the west as the + direction. If one wishes to employ co-ordinate axes, with the east as positive direction, it is a simple matter to change the sign.

§ 21. In addition to the hourly values, the tables give the *Range* of the inequality, defined as the difference between the algebraically greatest and least of the mean *hourly* values. They also give the sum of the differences of the 24 hourly values from their arithmetic mean—*i.e.*, from the mean for the day—taken numerically.

The meaning assigned to the term "range" is carefully to be borne in mind, because there are a variety of usages, some of which are technically more exact. If we suppose for simplicity that the phenomena are absolutely identical for each day of a month, then the maximum and minimum for each day will occur at fixed times, which will also be the times of occurrence of the maximum and minimum in the mean diurnal inequality for the month. And if it happen in addition that the times of occurrence of the maximum and minimum each coincides with an exact hour G.M.T., then the meaning to be assigned to the term "range" has no ambiguity. In practice, however, the maximum and minimum in any single day seldom come at exact hours, so that if we confine ourselves to hourly values we do not as a rule hit on either the absolute maximum or absolute minimum, and the difference between the greatest and least of our hourly values as a rule is less than the amplitude proper of the diurnal movement. Again, the times of occurrence of the maximum and minimum really vary from day to day, so that the range we obtain by considering mean hourly values from a series of days tends to be less than the arithmetic mean of the ranges deduced from the individual days results. This last consideration doubly requires to be taken into account in connection with inequalities for the year or for a season.

TABLE XI.—Diurnal Inequality

Hour	Forenoon. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
January (1890 to 1900)	−0′·88	−0′·60	**−0′·45**	−0′·48	−0′·58	−0′·65	−0′·82	−1′·08	**−1′·12**	−0′·23	+1′·10	+2′·30
February ,,	−1·15	−1·02	−0·90	**−0·89**	−1·01	−1·14	−1·28	−1·43	**−1·45**	−0·51	+1·17	+2·67
March ,,	−0·98	**−0·93**	−1·08	−1·26	−1·29	−1·60	−2·24	**−3·33**	−3·31	−1·64	+1·33	+4·18
April ,,	−0·83	−0·92	−1·09	−1·40	−1·66	−2·38	−3·62	**−4·52**	−4·16	−2·06	+1·09	+4·33
May ,,	−0·74	−1·01	−1·22	−1·87	−3·04	−4·01	**−4·78**	−4·50	−3·21	−0·43	+2·64	+5·32
June ,,	−0·74	−0·98	−1·23	−2·00	−3·41	−4·60	**−5·02**	−4·79	−3·63	−1·23	+1·63	+4·42
July ,,	−0·80	−1·03	−1·34	−1·98	−3·29	−4·20	**−4·37**	−4·27	−3·21	−1·22	+1·49	+4·22
August ,,	−1·05	−1·28	−1·61	−2·02	−2·85	−3·71	**−4·37**	−4·09	−2·62	+0·07	+3·03	+5·61
September ,,	−1·19	−1·25	−1·53	−1·92	−2·10	−2·72	−3·45	**−3·68**	−2·46	+0·11	+3·02	+5·41
October ,,	−1·03	−0·94	**−0·91**	−1·00	−1·12	−1·41	−1·97	−2·83	**−2·84**	−1·15	+1·65	+3·84
November ,,	−0·82	−0·64	−0·51	**−0·47**	−0·66	−0·89	−1·03	−1·44	**−1·52**	−0·36	+1·44	+2·78
December ,,	−0·56	−0·30	−0·23	**−0·23**	−0·26	−0·46	−0·63	−0·77	**−0·80**	+0·01	+0·98	+1·83
Mean for year (1890 to 1900)	**−0·90**	−0·91	−1·01	−1·29	−1·77	−2·31	−2·80	**−3·06**	−2·53	−0·72	+1·71	+3·91
,, ,, (1890, 1899, 1900)	**−0·63**	−0·63	−0·76	−1·02	−1·45	−1·88	−2·32	**−2·56**	−2·05	−0·48	+1·61	+3·49
,, ,, (1892 to 1895)	**−1·12**	−1·18	−1·30	−1·60	−2·18	−2·86	−3·39	**−3·68**	−3·06	−0·98	+1·83	+4·45

TABLE XII.—Diurnal

Hour	Forenoon. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
January (1891 to 1900)	−0′·03	−0′·08	−0′·13	−0′·27	−0′·37	−0′·41	**−0′·43**	−0′·27	+0′·12	+0′·39	**+0′·55**	+0′·42
February ,,	−0·12	−0·14	−0·12	−0·17	−0·28	**−0·36**	−0·36	−0·24	+0·19	+0·49	**+0·65**	+0·51
March ,,	−0·28	−0·28	−0·28	−0·27	**−0·39**	−0·28	−0·24	+0·15	+0·71	**+0·99**	+0·98	+0·66
April ,,	−0·37	−0·36	−0·34	−0·32	−0·29	−0·27	−0·05	+0·40	+0·92	**+1·36**	**+1·36**	+0·93
May ,,	−0·36	−0·26	−0·21	−0·13	−0·01	+0·26	+0·70	+1·10	**+1·32**	+1·23	+0·93	+0·44
June ,,	−0·29	−0·24	−0·21	−0·17	−0·04	+0·26	+0·68	+1·03	**+1·27**	+1·26	+0·95	+0·51
July ,,	−0·34	−0·29	−0·29	−0·21	−0·08	+0·22	+0·56	+0·86	+1·20	**+1·38**	+1·06	+0·65
August ,,	−0·45	−0·37	−0·34	−0·27	−0·07	+0·17	+0·61	+1·13	**+1·45**	+1·43	+1·07	+0·46
September ,,	−0·44	−0·40	−0·31	−0·28	−0·19	+0·01	+0·42	+0·94	+1·34	**+1·43**	+1·04	+0·38
October ,,	−0·34	−0·29	−0·33	−0·34	**−0·44**	−0·41	−0·21	+0·24	+0·80	**+1·13**	+1·11	+0·78
November ,,	−0·14	−0·16	−0·21	−0·32	−0·43	**−0·50**	−0·38	−0·08	+0·42	+0·75	**+0·77**	+0·63
December ,,	+0·08	+0·06	−0·04	−0·15	−0·22	−0·27	**−0·30**	−0·20	−0·07	+0·16	**+0·33**	+0·28
Mean for year (1891 to 1900)	−0·26	−0·23	−0·23	−0·24	−0·23	−0·14	+0·08	+0·42	+0·81	**+1·00**	+0·90	+0·55

of Declination. (+ to West.)

Afternoon. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	Range.	Sum of 24 differences from mean.	
+2·89	+2·21	+1·37	+1·05	+0·67	+0·24	−0·14	−0·54	−0·93	−1·07	−1·18	−1·11	4·07	23·69	January (1890 to 1900).
+3·31	+3·27	+2·46	+1·19	+0·75	+0·44	+0·11	−0·33	−0·78	−0·98	−1·22	−1·31	4·76	30·77	February ,,
+5·49	+5·23	+3·64	+1·72	+0·45	+0·03	−0·35	−0·50	−0·68	−0·87	−0·95	−1·04	8·82	44·12	March ,,
+6·05	+5·72	+4·12	+2·46	+1·04	+0·14	−0·25	−0·22	−0·18	−0·33	−0·55	−0·85	10·57	49·97	April ,,
+6·14	+5·55	+3·96	+2·12	+0·84	+0·04	−0·18	−0·27	−0·22	−0·21	−0·37	−0·66	10·92	53·33	May ,,
+5·60	+5·50	+4·49	+3·20	+1·83	+1·00	+0·38	+0·17	+0·06	+0·14	−0·28	−0·55	10·62	56·88	June ,,
+5·76	+5·81	+4·61	+2·77	+1·24	+0·39	+0·17	+0·10	+0·07	+0·02	−0·27	−0·62	10·18	53·25	July ,,
+6·64	+5·78	+4·06	+1·96	+0·34	−0·39	−0·34	−0·38	−0·40	−0·60	−0·73	−0·95	11·01	54·88	August ,,
+6·08	+5·03	+3·14	+1·39	+0·32	−0·09	−0·19	−0·48	−0·61	−0·71	−0·87	−1·13	9·76	48·92	September ,,
+4·67	+4·12	+2·89	+1·40	+0·77	+0·38	+0·01	−0·34	−0·82	−1·02	−1·09	−1·17	7·51	39·37	October ,,
+3·23	+2·52	+1·59	+0·99	+0·52	+0·17	−0·13	−0·53	−0·90	−1·19	−1·13	−1·12	4·75	26·58	November ,,
+2·25	+1·79	+1·34	+0·74	+0·27	−0·01	−0·33	−0·68	−0·97	−1·09	−0·99	−0·90	3·34	18·42	December ,,
+4·84	+4·38	+3·14	+1·75	+0·75	+0·19	−0·10	−0·33	−0·53	−0·66	−0·80	−0·95	7·90	41·34	Mean for year (1890 to 1900).
+4·15	+3·54	+2·28	+1·13	+0·38	0·00	−0·18	−0·34	−0·47	−0·54	−0·59	−0·67	6·71	33·15	,, ,, (1890, 1899, 1900)
+5·69	+5·31	+3·98	+2·36	+1·09	+0·39	0·00	−0·35	−0·60	−0·71	−0·95	−1·14	9·37	50·20	,, ,, (1892 to 1895).

Inequality of Inclination.

Afternoon. 1.	2.	3	4.	5.	6.	7.	8.	9.	10.	11.	12.	Range.	Sum of 24 differences from mean.	
+0·16	+0·12	+0·17	+0·19	+0·10	−0·05	−0·12	−0·05	−0·05	−0·01	+0·02	+0·02	0·98	4·53	January (1891 to 1900).
+0·27	+0·15	+0·14	+0·24	+0·15	+0·02	−0·07	−0·19	−0·24	−0·19	−0·17	−0·17	1·01	5·63	February ,,
+0·30	+0·12	+0·03	+0·05	+0·07	−0·09	−0·26	−0·32	−0·28	−0·27	−0·36	−0·33	1·38	8·09	March ,,
+0·49	+0·17	−0·03	−0·17	−0·28	−0·42	−0·45	−0·50	−0·47	−0·47	−0·46	−0·42	1·86	11·30	April ,,
+0·11	−0·03	−0·20	−0·33	−0·51	−0·61	−0·73	−0·64	−0·62	−0·53	−0·51	−0·43	2·05	12·20	May ,,
+0·21	−0·09	−0·27	−0·34	−0·50	−0·65	−0·75	−0·68	−0·62	−0·52	−0·45	−0·36	2·02	12·35	June ,,
+0·28	−0·03	−0·34	−0·34	−0·49	−0·58	−0·67	−0·60	−0·59	−0·52	−0·44	−0·38	2·05	12·40	July ,,
+0·10	−0·08	−0·17	−0·22	−0·29	−0·45	−0·68	−0·69	−0·70	−0·54	−0·58	−0·48	2·15	12·80	August ,,
−0·07	−0·22	−0·15	−0·03	−0·11	−0·28	−0·54	−0·55	−0·54	−0·51	−0·53	−0·46	1·98	11·17	September ,,
+0·39	+0·11	+0·12	+0·16	−0·04	−0·22	−0·34	−0·37	−0·38	−0·35	−0·38	−0·39	1·57	9·67	October ,,
+0·45	+0·28	+0·19	+0·09	−0·13	−0·21	−0·27	−0·23	−0·17	−0·14	−0·11	−0·12	1·27	7·18	November ,,
+0·17	+0·19	+0·15	+0·06	−0·03	−0·11	−0·14	−0·08	−0·03	0·00	+0·03	+0·09	0·63	3·24	December ,,
+0·24	+0·06	−0·03	−0·05	−0·17	−0·30	−0·42	−0·41	−0·39	−0·34	−0·33	−0·29	1·42	8·12	Mean for year (1891 to 1900).

TABLE XIII.—Diurnal Inequality of Horizontal

	Forenoon.											
Hour	1.	2.	3.	4.	5.	6.	7.	8.	9.	10	11.	12.
January (1890 to 1900)	+ 2	+ 3	+14	+35	+50	+58	**+61**	+ 33	− 23	− 68	**− 92**	− 74
February ,, ...	+17	**+15**	+23	+23	+40	+49	**+52**	+ 30	− 32	− 89	**−115**	− 97
March ,, ...	+43	+45	+42	**+40**	+58	**+60**	+42	− 18	−109	−175	**−192**	−148
April ,, ...	+66	+58	+56	+53	+54	+51	+21	− 55	−150	−237	**−258**	−200
May ,, ...	+69	+46	+41	+29	+15	−23	−88	−166	−219	**−234**	−210	−142
June ,, ...	+44	+33	+31	+28	+14	−36	−98	−162	−213	**−233**	−205	−133
July ,, ...	+52	+40	+41	+31	+16	−30	−81	−145	−207	**−245**	−219	−156
August ,, ...	+73	+61	+55	+44	+22	−15	−81	−167	−232	**−252**	−210	−123
September ,, ...	+74	+66	+52	+47	+37	+ 5	−55	−140	−215	**−250**	−207	−103
October ,, ...	+53	**+48**	+53	+56	**+68**	+62	+36	− 29	−124	−195	**−203**	−155
November ,, ...	+18	+19	+28	+44	+61	**+68**	+49	+ 8	− 64	−127	**−132**	−107
December ,, ...	− 9	− 6	+ 6	+23	+34	+42	**+44**	+ 29	+ 2	− 38	**− 62**	− 52
Mean for year (1890 to 1900)	+42	**+36**	+37	+38	**+39**	+24	− 8	− 65	−132	**−179**	−175	−124
,, ,, (1890, 1899, 1900). . .	+33	**+26**	+30	+30	**+32**	+15	−10	− 56	−109	**−142**	−136	− 86
,, ,, (1892 to 1895)	+53	+49	**+48**	+50	**+53**	+35	− 6	− 77	−161	−222	**−225**	−168

TABLE XIV.—Diurnal Inequality of Vertical

	Forenoon.											
Hour	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
January (1891 to 1900)	− 3	− 6	− 8	− 7	−12	−13	−16	−21	−23	− 30	**− 32**	− 32
February ,, ...	+ 1	− 9	−10	− 8	−10	−11	−14	−15	−18	− 45	**− 54**	− 50
March ,, ...	+17	+12	+ 6	+ 5	+ 4	+ 3	+13	+ 8	−25	− 80	−123	**−133**
April ,, ...	+26	**+13**	+12	**+10**	+16	+20	**+27**	+14	−34	− 92	−154	**−171**
May ,, ...	+30	+26	**+25**	+30	**+38**	+37	+23	−12	−69	−144	−201	**−209**
June ,, ...	+ 6	**− 2**	+ 1	+ 4	**+14**	+ 7	− 1	−28	−69	−114	**−158**	−149
July ,, ...	+15	+ 3	**− 6**	− 3	+ 7	− 8	−16	−36	−73	−103	−154	**−165**
August ,, ...	+18	+11	**+ 5**	+10	+25	**+26**	+21	− 7	−55	−111	−143	**−150**
September ,, ...	+22	+19	+13	**+12**	+12	+18	**+18**	− 9	−55	−112	**−138**	−135
October ,, ...	+12	+ 6	+ 4	+ 6	− 3	− 1	+ 7	+ 5	−23	− 74	**−103**	−101
November ,, ...	0	0	− 2	− 4	− 5	−13	−18	−11	−18	− 45	**− 51**	− 40
December ,, ...	+ 4	+ 2	+ 1	+ 2	+ 3	− 1	− 5	− 9	−20	**− 33**	− 32	− 31
Mean for year (1891 to 1900)	+12	+ 6	**+ 3**	+ 5	+ 7	+ 5	+ 3	−10	−40	− 82	−112	**−114**

Force. (Unit $= 0 \cdot 1\gamma \equiv 1 \times 10^{-6}$ C.G.S.)

Afternoon.												Range.	Sum of 24 differences from mean.	
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.			
− 29	−10	−11	−15	+ 2	+ 19	+ **26**	+ 15	+ 12	+ 2	− **2**	− 1	153	657	January (1890 to 1900).
− 57	−24	−12	−14	− 2	+ 14	+ 24	+ 37	+ **40**	+31	+28	+27	167	892	February ,,
− 82	−30	+ 6	+16	+ 16	+ 36	+ 61	+ **64**	+ 57	+56	+58	+51	256	1505	March ,,
−125	−52	+ 8	+41	+ 66	+ 88	+ **99**	+ 98	+ 91	+83	+81	+72	357	2163	April ,,
− 76	−22	+23	+63	+103	+129	+**147**	+127	+114	+97	+91	+75	381	2349	May ,,
− 73	− 3	+47	+71	+102	+136	+**153**	+137	+122	+96	+76	+61	386	2307	June ,,
− 90	−15	+56	+80	+108	+128	+**138**	+129	+118	+99	+82	+63	383	2369	July ,,
− 52	+ 1	+33	+55	+ 72	+ 97	+**125**	+122	+120	+91	+93	+77	377	2273	August ,,
− 23	+18	+23	+23	+ 42	+ 65	+ 95	+ **98**	+ 93	+83	+87	+75	348	1976	September ,,
− 87	−36	− 9	− 5	+ 29	+ 51	+ 65	+ 65	+ **69**	+61	+63	+62	272	1684	October ,,
− 68	−32	−10	+ 3	+ 33	+ 41	+ **46**	+ 37	+ 29	+22	+15	+15	200	1076	November ,,
− 25	−19	− 9	+ 1	+ 6	+ 20	+ **22**	+ 13	+ 3	− 3	0	−**13**	106	481	December ,,
− 66	−19	+12	+27	+ 48	+ 69	+ **83**	+ 78	+ 72	+60	+56	+47	262	1536	Mean for year (1890 to 1900).
− 32	+ 4	+20	+21	+ 32	+ 48	+ **63**	+ 56	+ 50	+40	+38	+35	205	1144	,, ,, (1890, 1899, 1900).
−100	−39	+ 7	+34	+ 63	+ 91	+**104**	+100	+ 93	+79	+76	+64	329	1997	,, ,, (1892 to 1895).

Force. (Unit $= 0 \cdot 1\gamma \equiv 1 \times 10^{-6}$ C.G.S.)

Afternoon.												Range.	Sum of 24 differences from mean.	
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.			
− 17	+18	+31	+29	+ **33**	+ 31	+ 30	+ 21	+16	+10	+ 3	0	65	442	January (1891 to 1900).
− 38	0	+31	+46	+ **48**	+ 43	+ 34	+ 27	+21	+15	+ 8	+10	102	566	February ,,
−101	−44	+13	+58	+ **66**	+ 62	+ 59	+ 51	+46	+38	+26	+19	199	1012	March ,,
−140	−67	− 1	+37	+ 63	+ 79	+ **81**	+ 71	+60	+50	+43	+33	252	1314	April ,,
−154	−78	+ 1	+52	+ 91	+**105**	+100	+ 92	+75	+58	+48	+36	314	1734	May ,,
−110	−48	+ 4	+53	+ 90	+100	+**107**	+101	+78	+56	+36	+24	265	1360	June ,,
−136	−69	0	+60	+105	+**118**	+117	+103	+91	+68	+48	+31	283	1535	July ,,
−115	−47	+21	+61	+ 79	+ **83**	+ 69	+ 60	+52	+39	+28	+20	233	1256	August ,,
− 96	−37	+15	+53	+ **66**	+ 62	+ 64	+ 56	+49	+41	+34	+25	204	1161	September ,,
− 72	−38	+19	+45	+ **49**	+ 46	+ 42	+ 41	+43	+39	+28	+20	152	827	October ,,
− 16	+18	+39	+**42**	+ 37	+ 28	+ 23	+ 17	+14	+ 7	0	− 3	93	451	November ,,
− 17	+ 4	+16	+**25**	+ 23	+ 20	+ 15	+ 10	+ 6	+ 6	+ 6	+ 5	58	296	December ,,
− 84	−32	+16	+47	+ 63	+ **65**	+ 62	+ 54	+46	+36	+26	+18	179	948	Mean for year (1891 to 1900).

TABLE XV.—Diurnal Inequality of Northern

Hour	Forenoon. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
December (1890 to 1900)	0	−1	+9	+26	+37	+47	+52	+40	+14	−37	−75	−78
March ,,	+57	+58	+57	+58	+76	+83	+76	+35	−52	−141	−204	−207
June ,,	+54	+47	+49	+58	+67	+38	−14	−79	−146	−203	−222	−197
Mean for year (1890 to 1900)	+54	+48	+51	+57	+65	+59	+37	−14	−86	−160	−194	−180
,, ,, (1890, 1899, 1900)	+41	+35	+41	+45	+53	+44	+27	−13	−72	−128	−155	−137
,, ,, (1892 to 1895)	+68	+65	+66	+73	+85	+79	+48	−15	−105	−197	−244	−231

TABLE XVI.—Diurnal Inequality of Western

Hour	Forenoon. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
December (1890 to 1900)	−31	−17	−10	−5	−3	−11	−19	−31	−40	−11	+31	+77
March ,,	−37	−34	−42	−52	−48	−63	−101	−175	−201	−135	+10	+168
June ,,	−24	−40	−53	−93	−169	−244	−284	−292	−248	−132	+22	+185
Mean for year (1890 to 1900)	−33	−36	−40	−54	−78	−110	−145	−175	−168	−90	+35	+162
,, ,, (1890, 1899, 1900)	−22	−24	−30	−43	−64	−91	−121	−147	−137	−67	+41	+152
,, ,, (1892 to 1895)	−41	−45	−52	−66	−95	−135	−174	−210	−203	−116	+26	+176

TABLE XVII.—Diurnal Inequality of Total

Hour	Forenoon. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
December (1890 to 1900)	0	0	+2	+11	+16	+15	+12	+3	−18	−45	−53	−49
March ,,	+32	+28	+22	+20	+27	+27	+28	+1	−65	−141	−186	−177
June ,,	+26	+14	+14	+16	+19	−9	−42	−91	−150	−200	−225	−190
Mean for year (1890 to 1900)	+27	+19	+17	+19	+22	+14	0	−34	−88	−145	−171	−153

Component. (Unit = $0{\cdot}1\gamma \equiv 1 \times 10^{-6}$ C.G.S.)

Afternoon. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	Range.	Sum of 24 differences from mean.	
− 60	− 47	−30	−11	+ 1	+ 19	+ **26**	+ 23	+ 18	+14	+16	+ 2	130	683	December (1890 to 1900).
−165	−111	−52	−12	+ 8	+ 34	+ 64	+ 69	+ 65	+67	+**70**	+65	290	1886	March ,,
−160	− 90	−26	+17	+69	+114	+**140**	+128	+116	+90	+77	+67	362	2268	June ,,
−140	− 87	−38	− 2	+34	+ 63	+ **81**	+ 80	+ 77	+68	+66	+60	275	1801	Mean for year (1890 to 1900).
− 96	− 52	−17	+ 2	+25	+ 46	+ **63**	+ 59	+ 55	+47	+46	+44	218	1343	,, ,, (1890, 1899, 1900).
−185	−121	−56	− 5	+43	+ 81	+ 99	+**101**	+ 98	+87	+87	+79	345	2318	,, ,, (1892 to 1895).

Component. (Unit = $0{\cdot}1\gamma \equiv 1 \times 10^{-6}$ C.G.S.)

Afternoon. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	Range.	Sum of 24 differences from mean.	
+**107**	+ 85	+ 65	+ 38	+ 15	+ 5	−10	−31	−48	−**56**	−50	−50	163	846	December (1890 to 1900).
+255	+**257**	+187	+ 92	+ 28	+12	0	− 6	−18	−28	−31	−**38**	458	2018	March ,,
+263	+**279**	+242	+184	+123	+91	+65	+50	+39	+36	+ 9	−10	571	3177	June ,,
+**226**	+217	+163	+ 97	+ 52	+30	+20	+ 6	− 5	−16	−24	−34	401	2016	Mean for year (1890 to 1900).
+**201**	+181	+122	+ 64	+ 29	+14	+ 9	0	− 9	−15	−19	−24	348	1626	,, ,, (1890, 1899, 1900).
+**259**	+258	+204	+130	+ 74	+47	+31	+12	− 3	−13	−26	−37	469	2433	,, ,, (1892 to 1895).

Force. (Unit = $0{\cdot}1\gamma \equiv 1 \times 10^{-6}$ C.G.S.)

Afternoon. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	Range.	Sum of 24 differences from mean.	
− 25	− 4	+11	+23	+ 24	+ **26**	+ 22	+ 14	+ 7	+ 4	+ 5	0	79	389	December (1890 to 1900).
−120	−52	+14	+60	+ 65	+ 69	+ **77**	+ 71	+ 63	+54	+44	+37	263	1480	March ,,
−130	−45	+21	+77	+121	+144	+**158**	+145	+120	+92	+65	+48	383	2162	June ,,
−103	−37	+19	+54	+ 77	+ 87	+ **89**	+ 80	+ 70	+56	+46	+35	260	1462	Mean for year (1890 to 1900).

Declination.

§ 22. A remark applicable to most if not all of the diurnal inequalities is that turning points, whether maxima or minima, which occur near noon are much less variable in the time of their appearance than those which occur early or late in the day. A particularly good instance of this is afforded by the chief maximum in D. It will be seen in Table XI. that the largest hourly value appears at 1 P.M. in 11 out of the 12 months. The uniformity in the time of appearance of the maximum is perhaps even better shown in the curves in fig. 2, which represent the D inequality for December, March and June. In the 6 months, October to March, the D inequality shows two distinct maxima and minima, and in every month of the year, with the exception of September, Table XI. shows at least a suggestion of a second maximum and minimum.

There is in all months a well marked morning minimum which, except in January and December, is the chief minimum of the day. Its time of occurrence shows a pronounced seasonal change, varying from 7 A.M. in summer to 9 A.M. in winter. This is well brought out in fig. 2, which also shows the rapid swing from east to west between 9 A.M. and 1 P.M. so characteristic of the declination needle. It would require a very large number of years' records, and possibly the employment of greater sensitiveness, to settle definitely the times of occurrence of the second maximum and minimum, and to place their occurrence or non-occurrence in summer in a position of certainty.

In the mean inequality for the year, we have the somewhat curious phenomenon of a maximum and minimum presenting themselves at adjacent hours, viz., 1 A.M. and midnight. As this occurs in the inequalities for the two sub-groups of years, as well as in that for the whole period, it is presumably a natural phenomenon.

It will be noticed that though the amplitudes of the mean inequalities for the year are widely different for the two groups of years, the hours of occurrence of the maxima and minima are alike. A minute comparison of the figures shows, however, a slight tendency for definite features of the inequality to be later in the day in the case of the second group of years (sun-spot maximum). More definite evidence in this point is adduced later (see § 46).

The month of the year in which the amplitude of the inequality is least is unmistakably December, but the ranges for January, February and November are also very small. There is a very rapid increase of range in March, and correspondingly rapid decrease in October, whilst in the six months April to September there is comparatively little change. The sum of the 24 hourly differences, however, is decidedly less for April and September than for the four mid-summer months. The range appears slightly larger in May and August than in June and July, so that the annual variation of the range seemingly presents two nearly equal maxima, respectively in early and late summer, with an intermediate inconspicuous minimum at midsummer, and a very prominent minimum at midwinter.

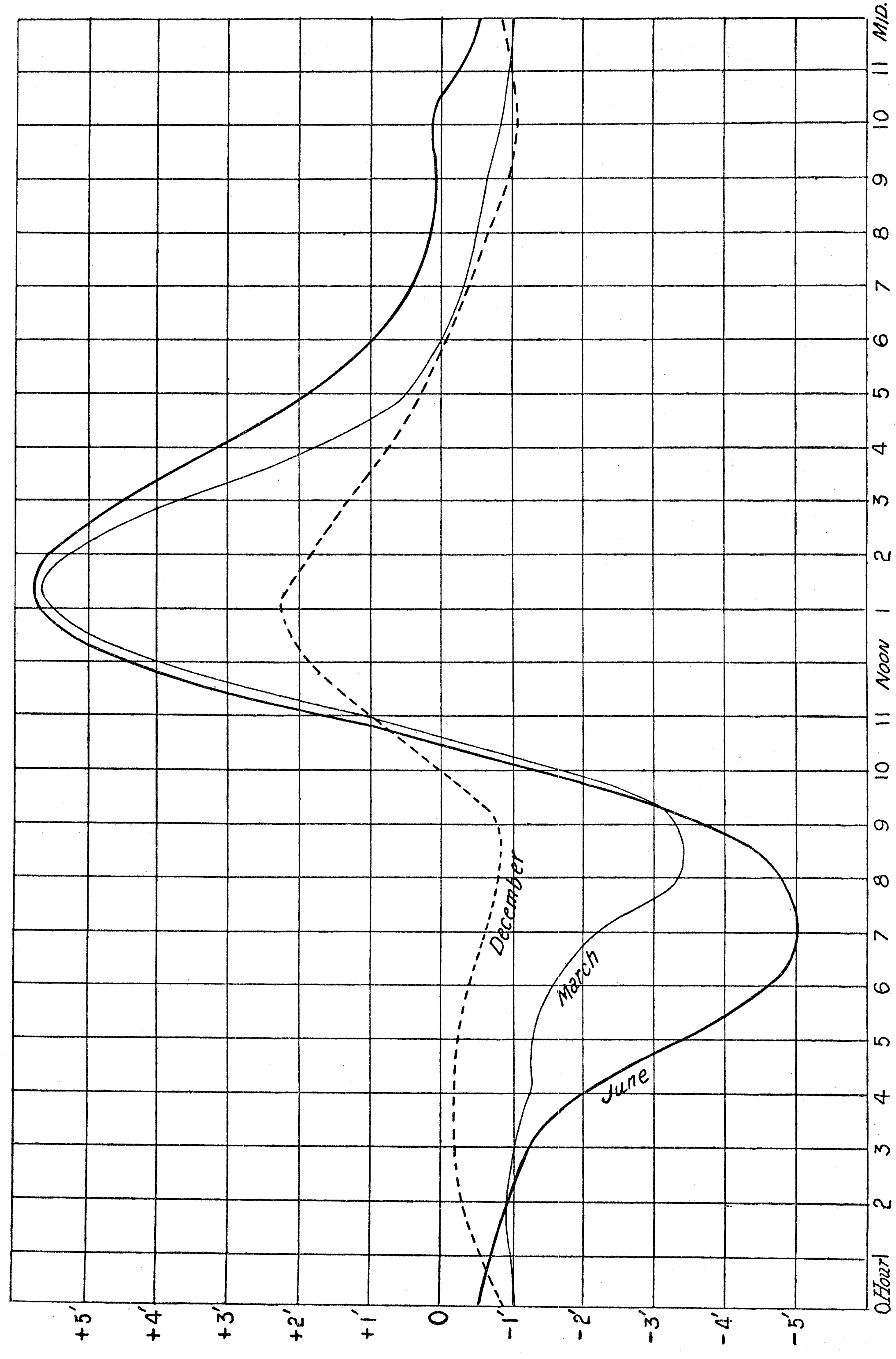


Fig. 2. Diurnal inequality of declination.

Inclination.

§ 23. The most distinct phenomenon in the diurnal inequality (see Table XII. and fig. 3) is the maximum between 9 and 11 A.M., its occurrence being earlier in summer than in winter. It is difficult to speak with certainty as to the number of maxima and minima in several months. In the 5 months April to August, there appears to be only one maximum and minimum, the latter occurring between 7 and 9 P.M.

In most of the other months there are at least two distinct maxima and minima, and sometimes suggestions of a third pair (see fig. 3). In the six months October to March, the principal minimum of the day is found in the morning between 5 and 7 A.M.

In the late evening the value is so nearly stationary in most months, that a very long series of observations might be necessary to elucidate the exact details.

The mean inequality for the year shows clearly only one maximum (at 10 A.M.), and one minimum (at 7 P.M.); but from 1 to 5 A.M. the value is practically stationary.

As regards the range, the phenomena are similar to those noted in the declination, but the existence of two maxima in summer with an intervening minimum is far from clearly shown. The minimum in December is very strongly marked.

The sum of the 24 differences also shows the December minimum clearly, but the value increases uniformly, though slightly from May to August.

Horizontal Force.

§ 24. Here two distinct maxima and minima appear in the 6 winter months, October to March, but only one is *clearly* shown in the summer months. In the mean inequality for the year two maxima and minima are distinctly recognisable.

Of the turning points the forenoon minimum is the best marked (*cf.* fig. 4). Its time of occurrence varies only from about 10 to 11 A.M., being earlier in summer than in winter. The afternoon maximum is less prominent, especially in winter, but is on the whole fairly definite. In summer and at midwinter it appears about 7 P.M., but in some of the equinoctial and winter months it is an hour, or even two, later.

The range and the sum of the 24 hourly differences from the mean both show a conspicuous minimum in December. In the 4 midsummer months it is now the sum of the 24 differences that shows two maxima and an intermediate badly-defined minimum. The range shows only one maximum, in June. The variation, however, in both cases is so small, from May to August, that the exact nature of the phenomena remains open to doubt.

Vertical Force.

§ 25. Here the principal minimum presents itself in the late forenoon. In the summer months the smallest hourly value usually occurs at noon; in the winter months it occurs earlier, usually at 11.

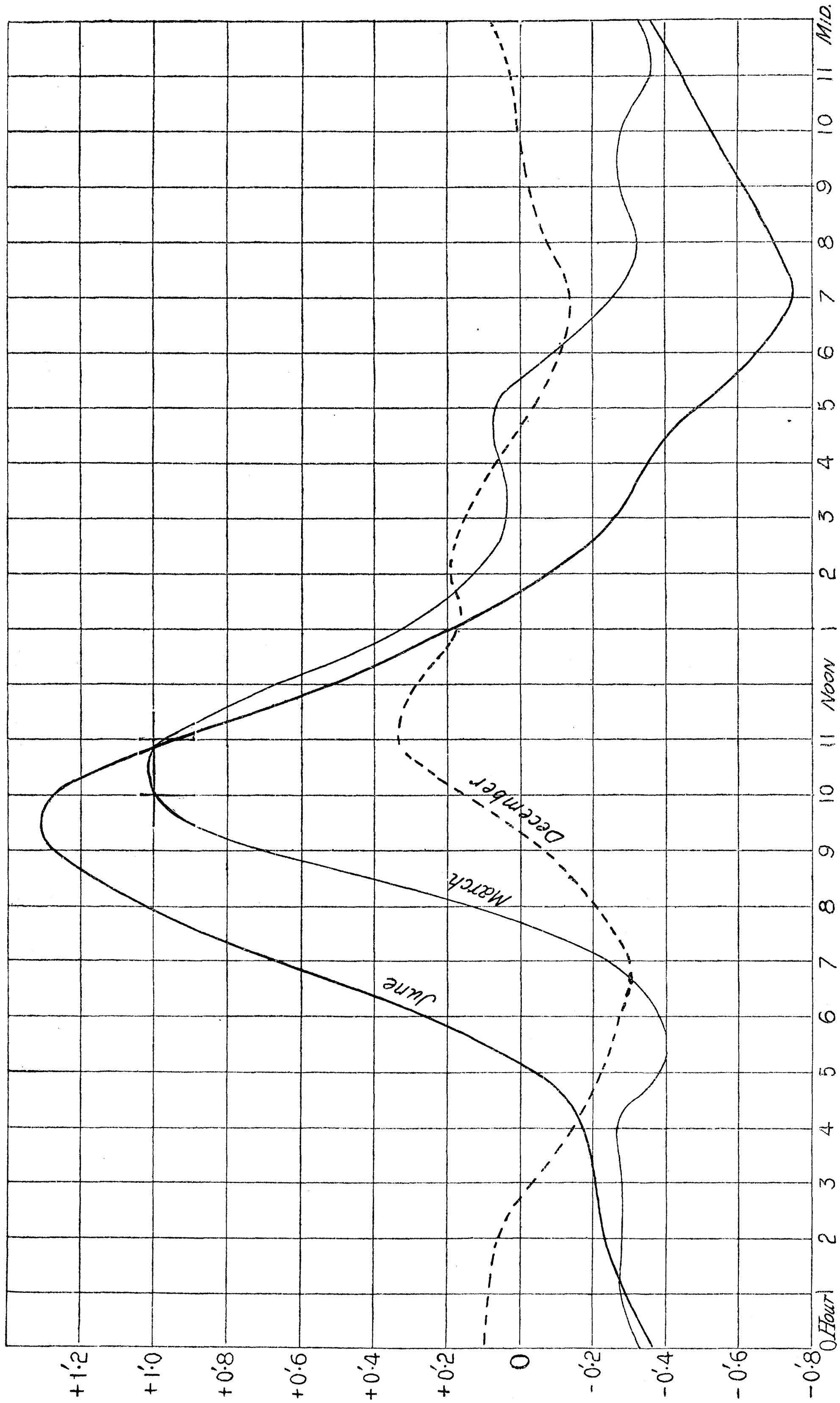


Fig. 3. Diurnal inequality of inclination.

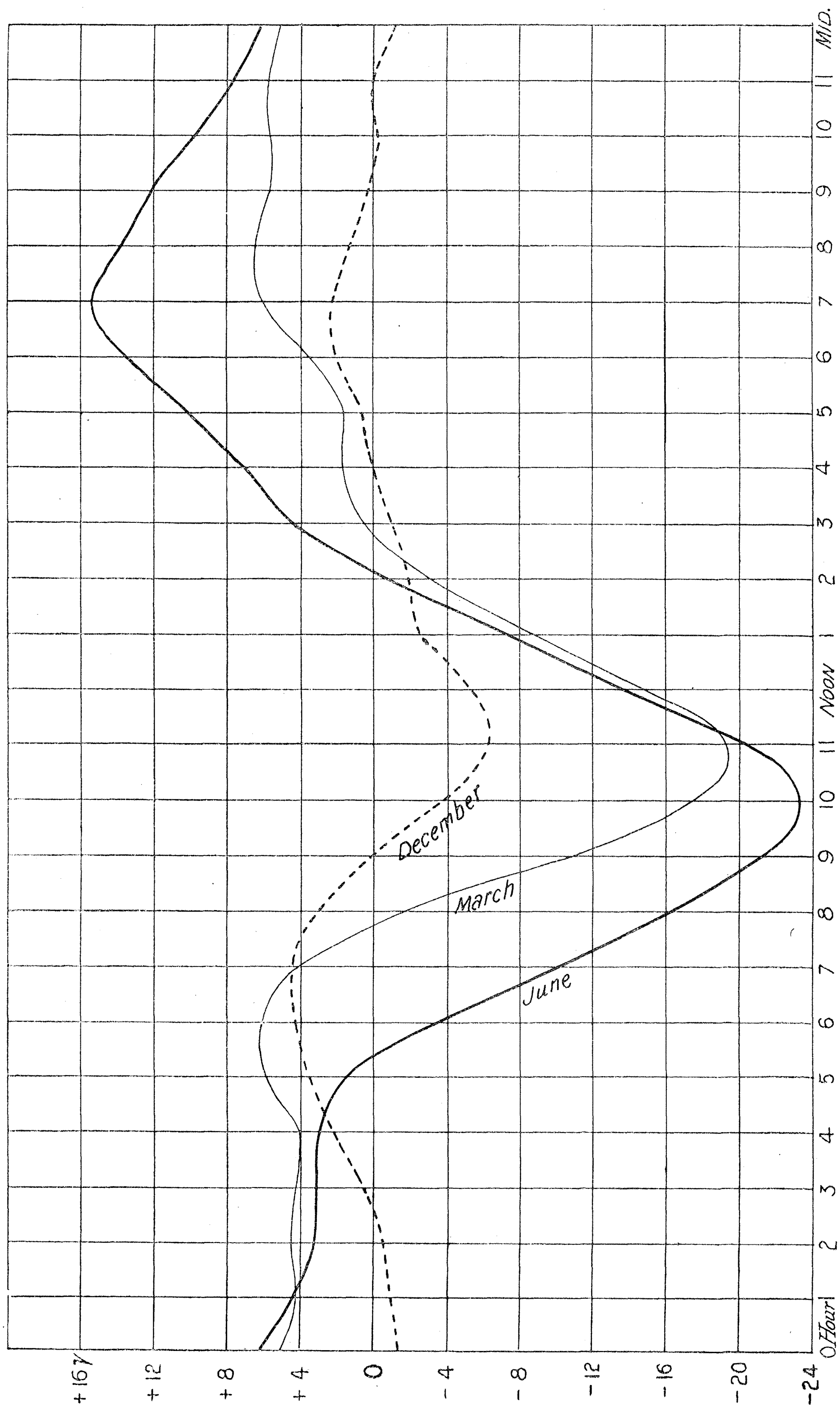


Fig. 4. Diurnal inequality of horizontal force.

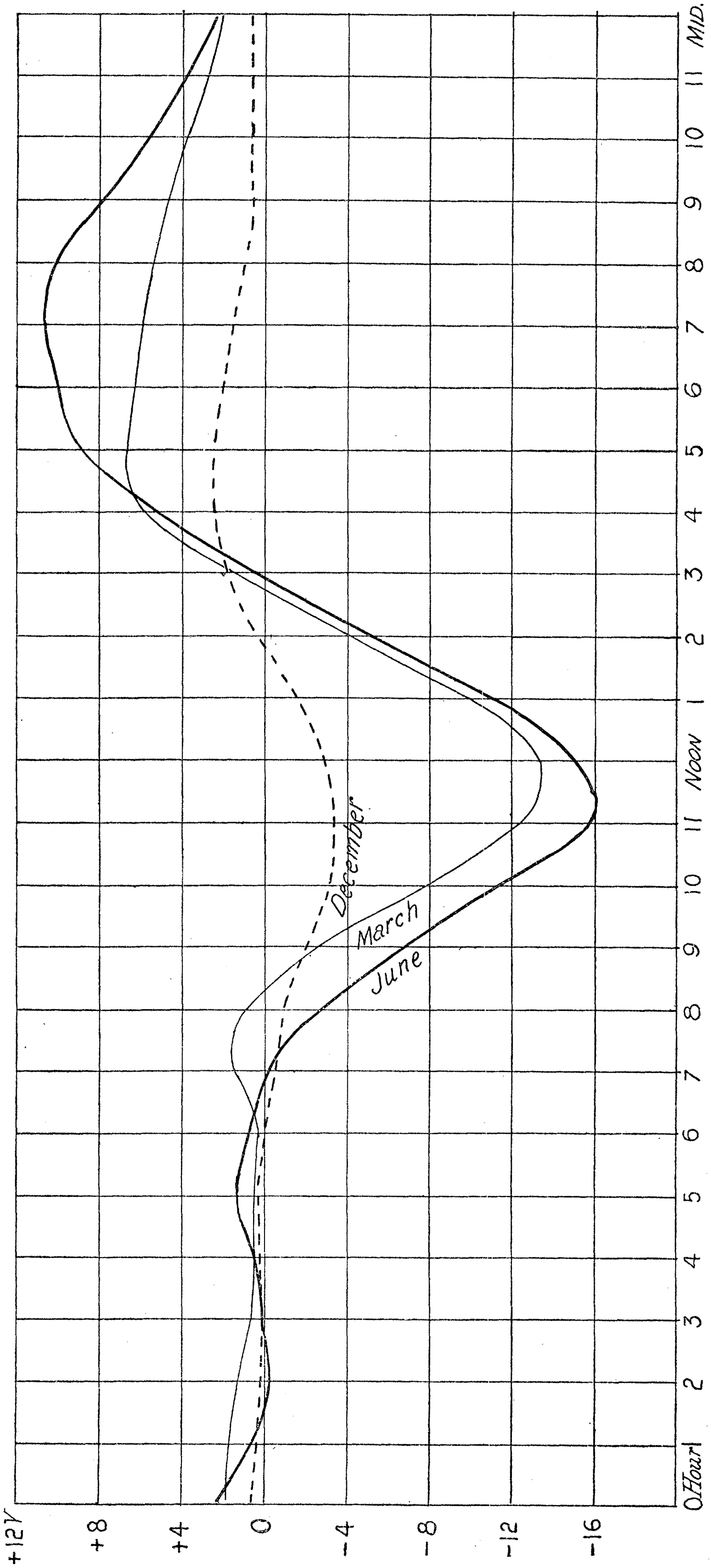


Fig. 5. Diurnal inequality of vertical force.

The principal maximum is found in the afternoon; the time varies from 4 to 7 P.M., and, as with the minimum, is earlier in winter than in summer (see fig. 5).

Normally there would seem to be 2 maxima and minima, the second maximum and minimum occurring in the early morning within a few hours of one another. This is shown, though faintly, even in the mean inequality for the year. In the winter months, however, the values in the earlier morning hours become so nearly stationary that the true nature of the phenomena is uncertain.

The ranges and the sum of the 24 hourly differences from the mean both show two maxima in summer, in May and July, the intervening minimum being more clearly marked than in the other elements. The winter minimum in December, on the other hand, is somewhat less clearly marked than in H or I.

Northerly and Westerly Components.

§ 26. All the months included in Table XV. show 2 maxima and minima in the diurnal inequality of N, and the same is true of the mean inequality for the year (*cf.* fig. 6). As with H, the most conspicuous turning point is the forenoon minimum; but its time of occurrence is an hour or so later than in H, varying from 11 A.M. to noon. In December and March the forenoon maximum is the larger; but in June, and in the mean inequality for the year, the afternoon maximum—occurring usually at 7 or 8 P.M.—is decidedly the larger of the two. The early morning minimum, though not very conspicuous, seems to have a very uniform time of occurrence.

The phenomena met with in the case of W (Table XVI., fig. 6) naturally resemble those met with in D. The maximum at 1 or 2 P.M., and the minimum at 8 or 9 A.M., are somewhat later than the corresponding turning points in the declination. In June there is no trace of a second maximum and minimum, and in the mean inequality for the year their presence is at least doubtful. In December the evening minimum is the more prominent of the two.

Total Force.

§ 27. The diurnal inequality of T (see Table XVII., figs. 6 and 7) shows 2 maxima and minima, of which the afternoon maximum at 6 or 7 P.M. and the forenoon minimum at about 11 A.M. are the most important. The early morning maximum and minimum occur pretty close together, and their difference is generally small.

The T inequality curves appear exceptionally regular, which is hardly what one would have expected, considering that variations in both H and V are involved.

From a mathematical standpoint, T is most naturally regarded as the resultant of W, N, and V, and so mean inequalities for the year for these elements are shown alongside of the corresponding inequality for T in fig. 6.

It will be observed in Tables XI. to XVII. that the range and the sum of the 24 differences in March generally resemble closely the corresponding quantities from the

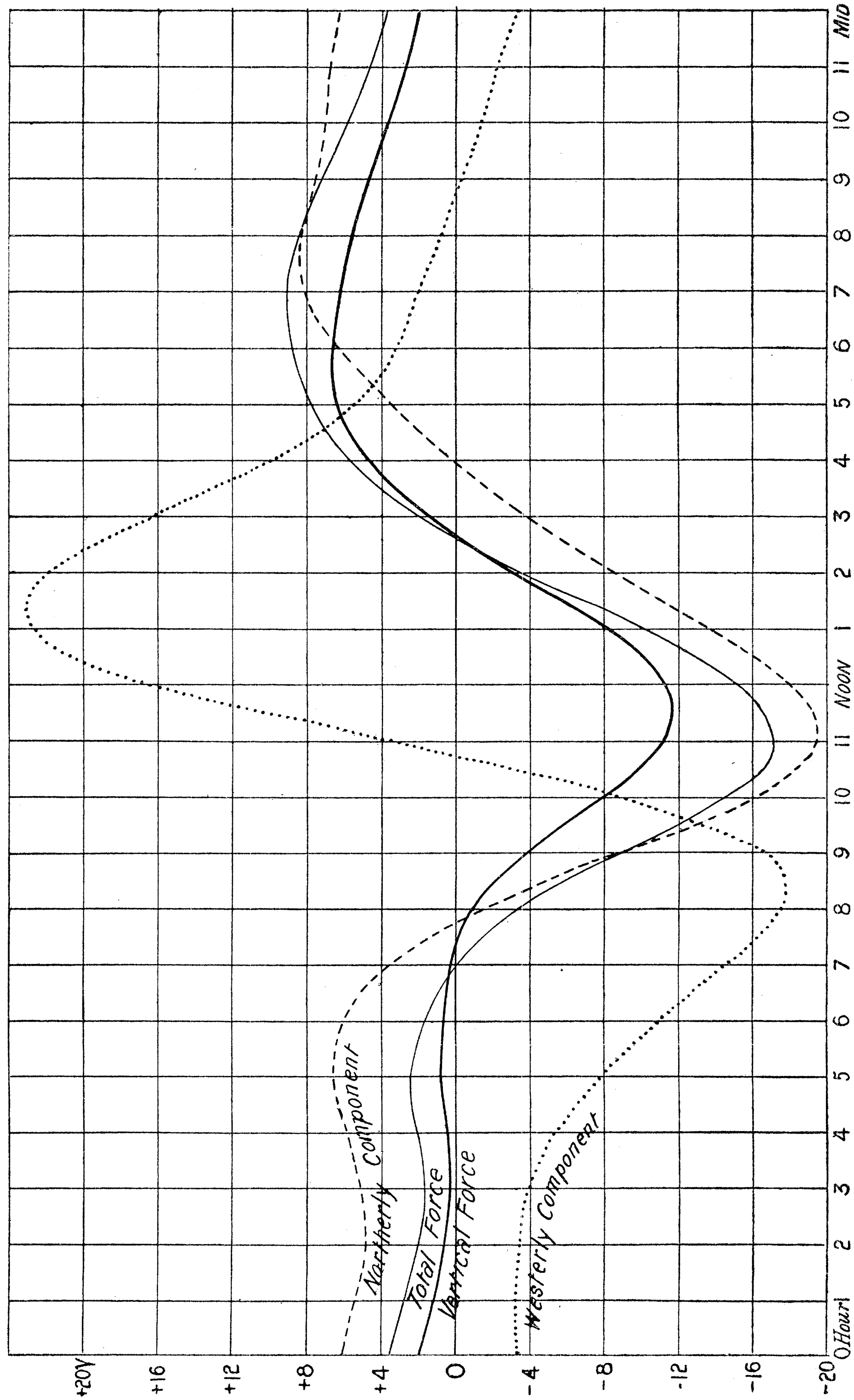


Fig. 6. Mean diurnal inequality for year.

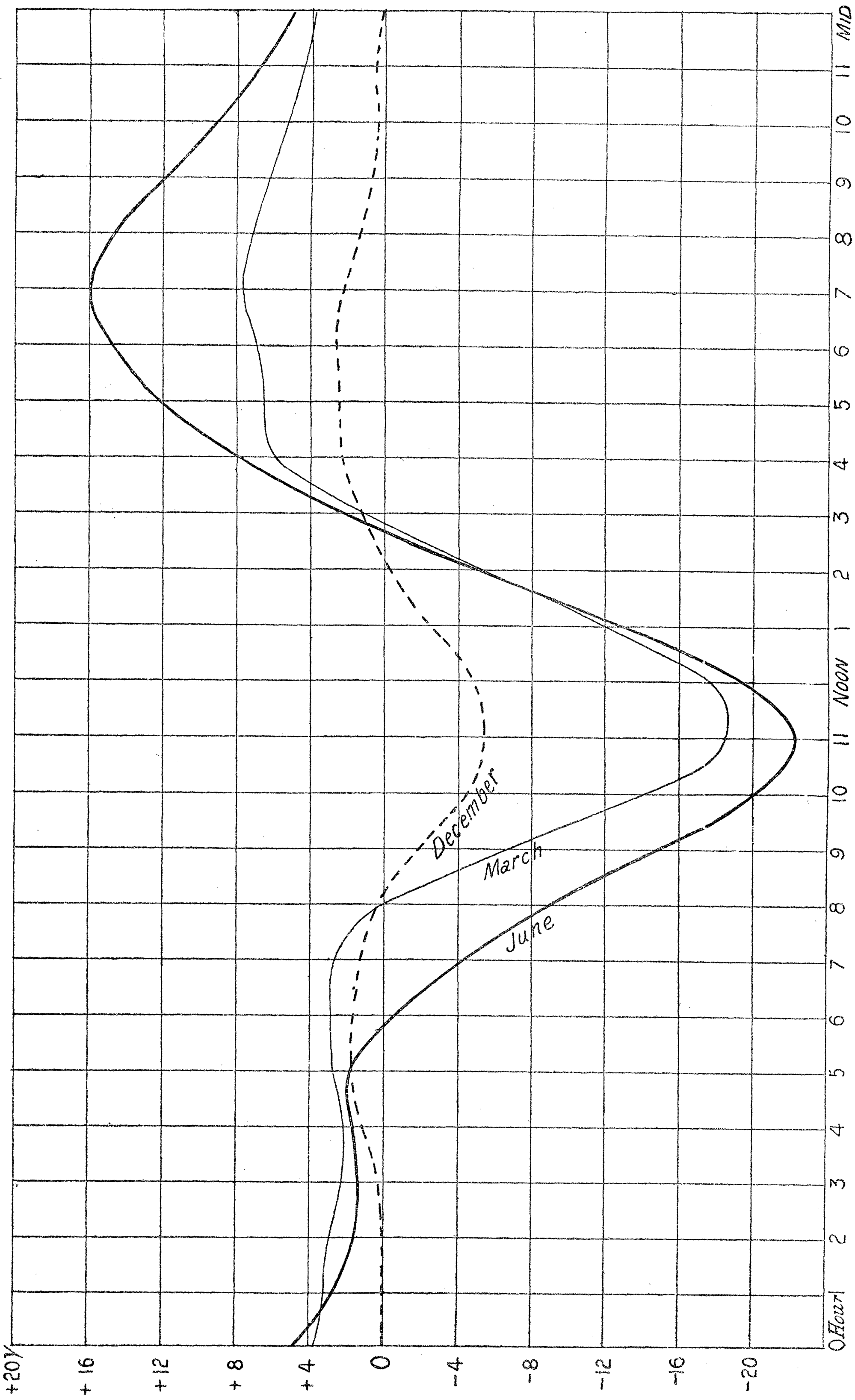


Fig. 7. Diurnal inequality of total force.

mean inequality for the year. In fact, the data from March give a better general idea of the phenomena than those from any other month.

§ 28. To facilitate comparison of the law of variation throughout the year of the range and the sum of the 24 differences in the diurnal inequalities of D, I, H, and V, I give a synopsis of the whole of the data in Table XVIII. Here each quantity dealt with is expressed as a percentage of its mean value for the 12 months. This enables us to see more readily points of agreement and difference. It will be observed that there is a close resemblance between the annual variations in D, I, and H, and that the mean of the results from these three elements for the range and for the sum of the 24 differences are very similar. On the whole, the sum of the 24 differences has the more accentuated annual variation.

The annual variation seems somewhat greater in V than in the other elements, the maximum in May being exceedingly prominent.

TABLE XVIII.—Monthly Relative Values. (100 ≡ Mean from 12 Months.)

	Range.					Sum of 24 differences from mean.				
	D.	I.	H.	V.	Mean of D, I, H.	D.	I.	H.	V.	Mean of D, I, H.
January	51	62	54	35	55	57	49	40	44	49
February	59	64	59	55	61	74	61	54	57	63
March	110	87	91	108	96	106	88	92	102	96
April	132	118	127	136	126	120	123	132	132	125
May	136	130	135	170	134	128	132	143	174	134
June	132	128	137	143	132	136	134	140	137	137
July	127	130	136	153	131	128	135	144	154	136
August	137	136	134	126	136	132	139	138	126	136
September	122	125	123	110	123	117	121	120	117	119
October	93	99	96	82	96	94	105	102	83	100
November	59	80	71	50	70	64	78	66	45	69
December	42	40	37	31	40	44	35	29	30	36

§ 29. Figs. 2 to 7 should give an adequate pictorial idea of the general character of the diurnal inequality in individual elements.

A more complete idea of the phenomenon as a whole can be obtained from two other types of curves. The first type, commonly known as the "vector diagram," has been used by LLOYD, AIRY and others. It represents the variation throughout the 24 hours of the disturbing force to which may be ascribed the changes in the horizontal component of magnetic force. The radius vector from a fixed point represents the horizontal component of the disturbing force in magnitude and direction. When inequalities for N and W have been calculated, the simplest way to draw the diagram is to take the hourly values of the N and W inequalities as co-ordinates, employing rectangular axes, one in, the other perpendicular to, the astronomical meridian. This has been done in drawing the vector diagrams for December, June, March and the whole year—all for the entire period 1890 to 1900—which are represented in figs. 8 to 11.

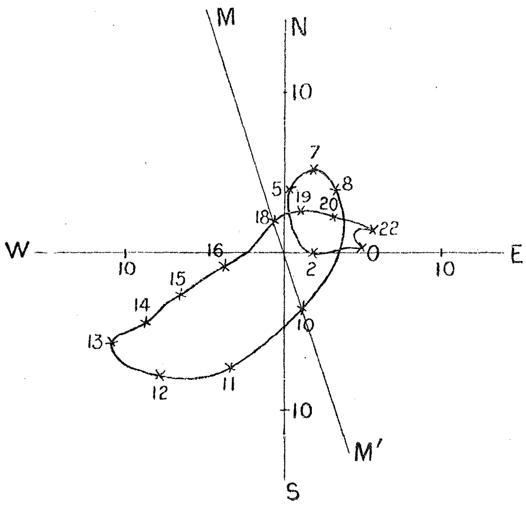


Fig. 8. December.

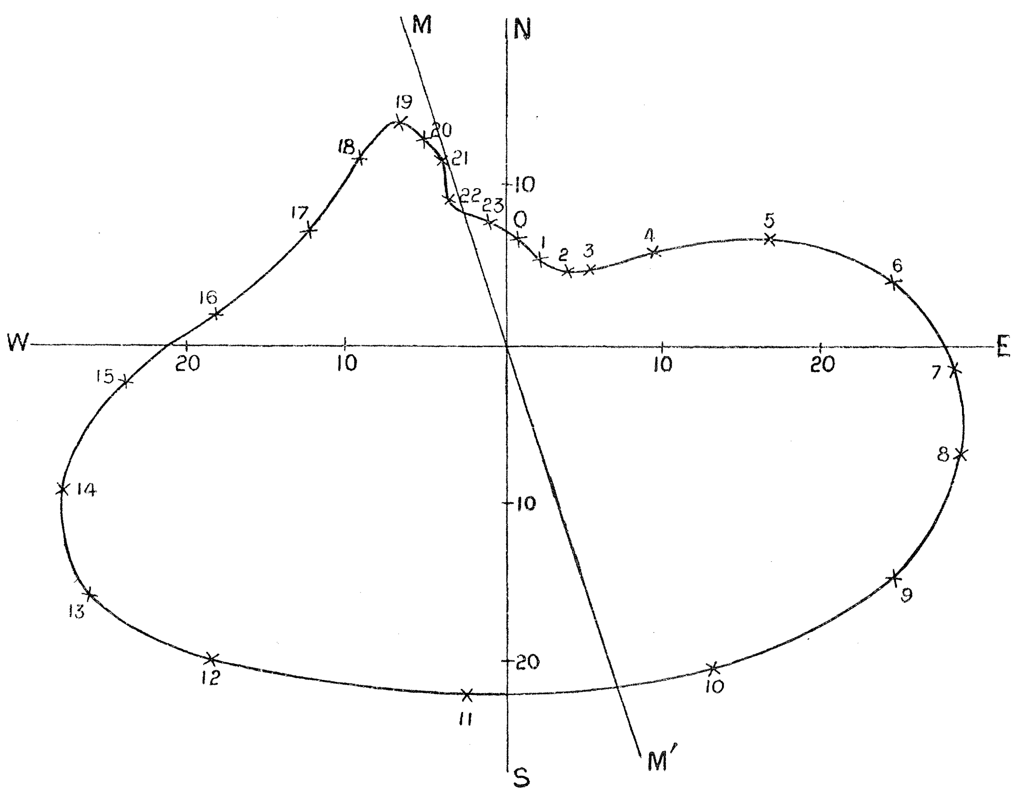


Fig. 9. June.

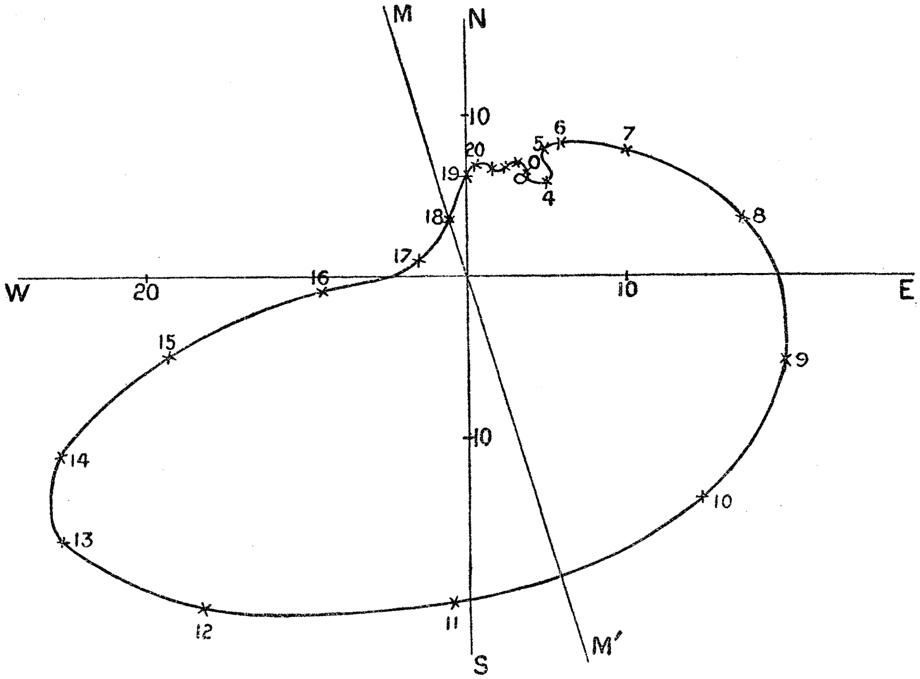


Fig. 10. March.

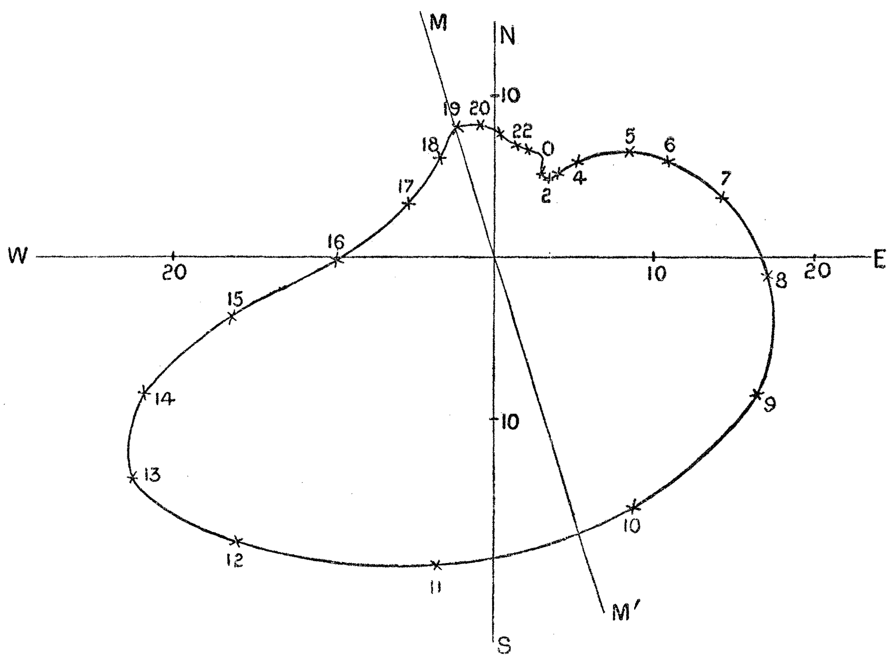


Fig. 11. Year.

In all the figures, NS represents the astronomical meridian, MM′ the mean magnetic meridian for the period, the two being inclined at an angle of 17° 18′·8. The hours to which the individual observations refer are shown in the figures, all being reckoned from midnight as 0.

When N and W inequalities are not available, the vector diagram can be readily constructed by taking axes in and perpendicular to the magnetic meridian, employing for co-ordinates δH and δD, the latter converted into absolute measure. This was the method which I employed in the ‘B. A. Report’ for 1895.

The difference between the vector diagrams for December and June is very striking, the area described by the radius vector in the latter curve being about 16 times that described in the former. The diagram for March pretty closely resembles that for the whole year, but encloses a somewhat larger area. In December the portion of the curve answering to the hours near midnight forms a regular closed loop—also seen in other midwinter months. The March curve shows a very tiny loop, and forms a transition to the June curve, where no loop appears, though the portion of the curve answering to the hours from 7 or 8 P.M. to 3 A.M. is still markedly indented.

The longest radius vector, and so the largest value of the disturbing force, answers to about 1 P.M. in all the curves. The shortest radius vector answers either to about 2 A.M. or 5 P.M. In all the curves the radius vector passes through astronomical south between 10 and 11 A.M., the time of crossing being somewhat earlier in December than in March or June.

§ 30. The second form of curve, illustrated by figs. 12 to 15, shows the change throughout the day in the direction of an imaginary magnet freely suspended as described in § 8. The curve may be supposed drawn by a style carried by the dipping end of the magnet, on a plane perpendicular to the mean position of the magnet throughout the day. MM′ is the trace of the *magnetic* meridian in this plane.

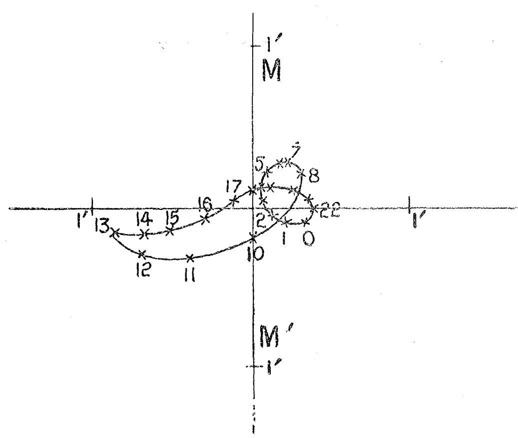


Fig. 12. December.

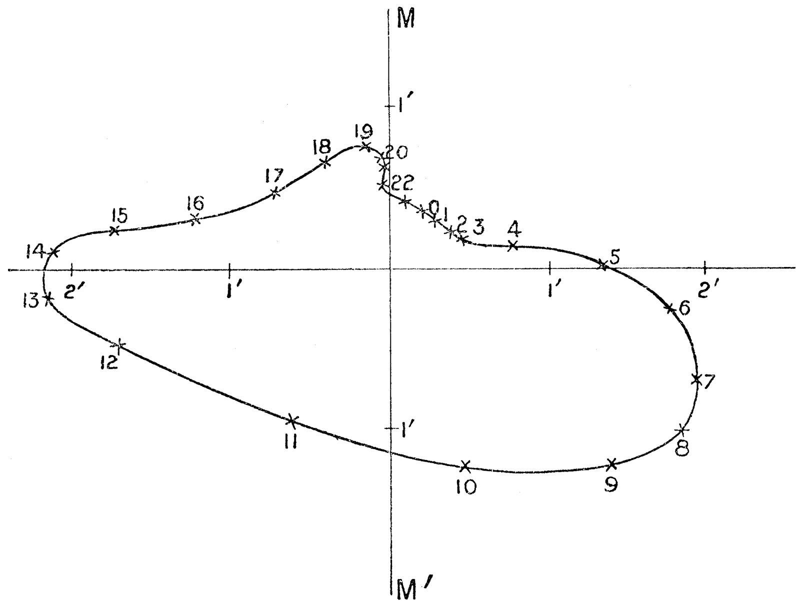


Fig. 13. June.

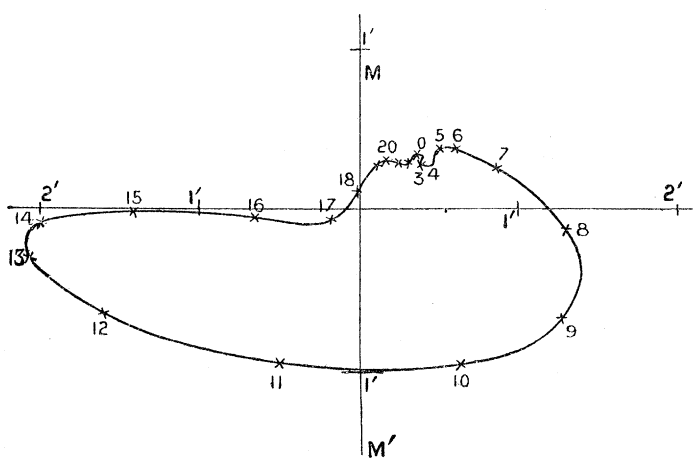


Fig. 14. March.

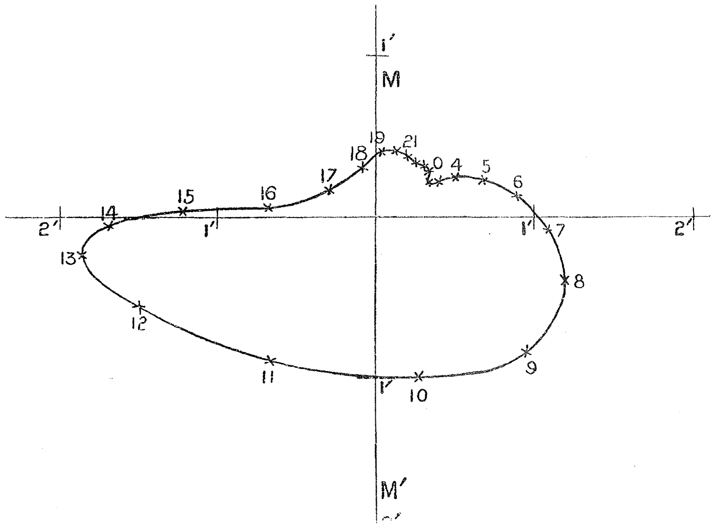


Fig. 15. Year.

The curve is really drawn from Cartesian co-ordinates, δI parallel to MM′, and cos I δD (or here ·385 δD) perpendicular to MM′. Increasing I answers to movement towards the foot of the paper, increasing D to movement towards the reader's left hand. The hours are counted and marked as in figs. 8, 9, 10 and 11, to which the figures now under discussion bear a general resemblance.

The angular distance of the needle from its mean position for the day is seen to be greatest about 1 P.M., though in June the distance at 8 A.M. is nearly as great. The approach to the mean position is closest shortly after midnight, or else about 5.30 P.M.

The movement perpendicular to the magnetic meridian is considerably larger than that in the meridian. The curves for March and for the year are oriented, so to speak, approximately magnetic east and west. The orientations of the June and December curves are almost equally inclined to this direction, but on opposite sides.

The curves of the second type were employed by the late Admiral CAPELLO in a paper in the 'B. A. Report' for 1898, p. 750, dealing with diurnal inequalities at a variety of stations. They were also used by Sir A. RÜCKER in a Rede Lecture to illustrate the actual motion of a freely suspended needle on a magnetic quiet day, showing the average non-cyclic effect experienced at Kew. In the present case the curves of both types answer to the strictly *periodic* part of the diurnal motion.

Analysis of Diurnal Inequality in Fourier Series.

§ 31. The diurnal inequality of an element may be expressed in either of the two equivalent forms

$$a_1 \cos t + b_1 \sin t + a_2 \cos 2t + b_2 \sin 2t + \ldots,$$

$$c_1 \sin (t + \alpha_1) + c_2 \sin (2t + \alpha_2) + \ldots$$

where t represent G.M.T. counted from the first midnight of the day, an hour being taken as equivalent to 15°; whilst a_1, b_1, c_1, α_1 . . . are constants. Between the constants of the two expressions there exist the relations

$$\alpha_n = \tan^{-1}(a_n/b_n), \quad c_n = \sqrt{a_n^2 + b_n^2}.$$

The constants with suffix n occur in terms in which the period is $24/n$ hours. The periods of the terms in the above expressions are respectively 24, 12, 8, 6 hours. In a previous paper dealing with Kew data I explained that terms with periods shorter than 6 hours were so small as to be of little account, and similar conclusions have been reached elsewhere. Thus attention will be confined to the first four periods. The values of a_n, b_n have been found by calculation from the well known formulæ; the values so obtained are, it should be noticed, independent of whether terms of the higher orders are neglected or not.

The data employed in the calculations refer to G.M.T. If results are wanted relating to local solar time, all that is necessary is the application of a table of corrections to the values found for the angles α_1, α_2, &c. In the case of α_1, the correction simply represents the angular equivalents of the equation of time and the longitude of the place of observation. The corrections to α_2, α_3, and α_4 are obtained by multiplying those for α_1 by 2, 3, and 4 respectively. The corrections for the individual months and seasons of the year applicable to Kew are given in Table XIX. They represent the mean of the results given in the 'Greenwich Magnetical and Meteorological Observations' for the four years 1892 to 1895, with the addition required to allow for the longitude of Kew, viz., 19′ W. The corrections to α_1—omitting of course the 19′—are in close agreement with corresponding results given by General STRACHEY,* there being sensible differences only in August and September. In these months, however, they are in good agreement with results given more recently by ANGOT.†

There are probably for a mean year errors of 1′ in the values of α_1 in the table, and these of course multiply up for α_2, α_3, and α_4; but the results are probably sufficiently exact for all practical purposes.

For the seasons, the use of local solar time fortunately makes but little difference; but in several months, notably February and November, the distinction between local solar time and G.M.T. is very appreciable.

* 'Phil. Trans.' A, for 1893, p. 646.

† 'Ann. du Bureau Central Météorologique de France,' 1899, "Mémoires," p. B. 98.

TABLE XIX.—Corrections to be applied to Angles in Fourier Expansions when G.M.T. replaced by Kew Solar Time.

Angle	α_1.	α_2.	α_3.	α_4.
	° ′	° ′	° ′	° ′
January	+2 42	+5 24	+ 8 6	+10 48
February.	+3 48	+7 36	+11 24	+15 12
March	+2 28	+4 56	+ 7 24	+ 9 52
April	+0 20	+0 40	+ 1 0	+ 1 20
May	−0 33	−1 6	− 1 39	− 2 12
June	+0 25	+0 50	+ 1 15	+ 1 40
July	+1 41	+3 22	+ 5 3	+ 6 44
August	+1 15	+2 30	+ 3 45	+ 5 0
September	−0 58	−1 56	− 2 54	− 3 52
October	−3 12	−6 24	− 9 36	−12 48
November	−3 20	−6 40	−10 0	−13 20
December	−0 40	−1 20	− 2 0	− 2 40
Year	+0 19	+0 38	+ 0 57	+ 1 16
Winter	+0 37	+1 14	+ 1 51	+ 2 28
Equinox	−0 21	−0 42	− 1 3	− 1 24
Summer	+0 42	+1 24	+ 2 6	+ 2 48

§ 32. Tables XX. to XXIII. give the values of the two sets of Fourier constants a, b, and c, α—using G.M.T.—for D, I, H, and V. Values are given for the inequalities for the several months, for the year as a whole, and for winter, equinox, and summer, each representing, as before, a group of 4 months.

The D and H results are based on the 11 years 1890 to 1900, the I and V results on the 10 years 1891 to 1900.

An a or b coefficient for a group of i months is the arithmetic mean of the a's or b's for the individual i months; but this is not in general true of the c's or α's. For instance, the arithmetic mean of the 12 values of c_1 for individual months of the year, in Table XX. is 2·371, the mean of the three values for c_1 for the three seasons is 2·356, whilst the value of c_1 from the mean inequality for the year is 2·322. The more variable the angle α throughout the season dealt with, the greater is the difference between the corresponding c for that season and the arithmetic mean of the c's for the individual months composing it.

In all probability it would require a very large number of years' data to supply smooth values for the coefficients of the 8-hour and 6-hour terms, and much weight cannot be assigned to apparent irregular fluctuations in the values of these coefficients from month to month. This is really one of the principal reasons for grouping the months into seasons. The four mid-winter months stand out rather noticeably from the two adjacent months March and October, so that separation into three seasons seems better for the present purpose than the more usual separation into two 6-month periods.

The values of the a, b, c constants were really calculated to at least one figure farther than appears in the tables, but the retention of the extra figure seemed more likely to mislead than to serve any useful purpose. It would have been more logical to have similarly restricted the values of the angles, but I have followed the example of the annual Greenwich Tables and given them to minutes. When it comes to comparing one season or one year with another, the minutes may possibly possess a real significance, but the uncertainties in the monthly data represent degrees rather than minutes.

TABLE XX.—Coefficients and Angles in Fourier Series

	Period, 24 hours.				Period, 12 hours.			
	a_1.	b_1.	c_1.	α_1.	a_2.	b_2.	c_2.	α_2.
January	−1′·18	−0′·61	1′·33	242° 40′	+0′·39	+0′·75	0′·84	27° 43′
February	−1·43	−1·06	1·78	233 38	+0·52	+0·86	1·01	31 3
March	−1·65	−1·66	2·34	224 55	+1·21	+1·59	2·00	37 12
April	−1·51	−2·29	2·75	213 26	+1·47	+1·88	2·39	38 0
May	−1·84	−2·46	3·07	216 47	+2·14	+1·50	2·61	55 3
June	−1·57	−3·03	3·42	207 24	+1·81	+1·66	2·45	47 23
July	−1·65	−2·75	3·21	210 58	+1·81	+1·57	2·39	49 0
August	−2·22	−2·23	3·15	224 55	+2·17	+1·37	2·57	57 52
September	−2·11	−1·83	2·79	229 2	+1·82	+1·16	2·16	57 27
October	−1·55	−1·44	2·12	227 12	+0·94	+1·31	1·61	35 34
November	−1·26	−0·72	1·45	240 26	+0·56	+0·85	1·02	33 33
December	−0·99	−0·32	1·04	251 53	+0·36	+0·68	0·77	27 54
Mean of 12 monthly values	−1·582	−1·700	—	—	+1·266	+1·264	—	—
Inequality for—								
Year	—	—	2·322	222 57	—	—	1·789	45 3
Winter	−1·217	−0·677	1·393	240 55	+0·458	+0·784	0·908	30 18
Equinox	−1·707	−1·805	2·484	223 24	+1·358	+1·486	2·013	42 25
Summer	−1·822	−2·620	3·191	214 49	+1·981	+1·523	2·499	52 27

TABLE XXI.—Coefficients and Angles in Fourier Series

	Period, 24 hours.				Period, 12 hours.			
	a_1.	b_1.	c_1.	α_1.	a_2.	b_2.	c_2.	α_2.
January	−0′·176	−0′·134	0′·221	232° 41′	+0′·219	−0′·016	0′·220	94° 17′
February	−0·283	−0·087	0·296	252 50	+0·171	+0·020	0·172	83 25
March	−0·477	+0·044	0·479	275 13	+0·212	−0·141	0·254	123 44
April	−0·651	+0·236	0·693	289 54	+0·305	−0·210	0·370	124 34
May	−0·569	+0·566	0·803	314 52	+0·099	−0·279	0·296	160 27
June	−0·560	+0·566	0·796	315 18	+0·185	−0·277	0·333	146 19
July	−0·588	+0·509	0·778	320 53	+0·179	−0·294	0·345	148 41
August	−0·656	+0·511	0·832	307 56	+0·083	−0·316	0·327	165 15
September	−0·594	+0·385	0·708	302 57	+0·055	−0·317	0·322	170 6
October	−0·566	+0·075	0·571	277 32	+0·249	−0·151	0·292	121 10
November	−0·363	−0·040	0·365	263 45	+0·312	−0·063	0·318	101 24
December	−0·081	−0·073	0·109	228 4	+0·189	+0·041	0·193	77 45
Mean of 12 monthly values	−0·4636	+0·2132	—	—	+0·1881	−0·1670	—	—
Inequality for—								
Year	—	—	0·5103	294 42	—	—	0·2515	131 36
Winter	−0·2256	−0·0835	0·2405	249 41	+0·2225	−0·0047	0·2225	91 13
Equinox	−0·5720	+0·1848	0·6010	287 54	+0·2052	−0·2046	0·2898	134 55
Summer	−0·5933	+0·5382	0·8009	312 12	+0·1365	−0·2916	0·3220	154 55

Expansion of Diurnal Inequality of Declination (1890 to 1900).

Period, 8 hours.				Period, 6 hours.				
a_3.	b_3.	c_3.	α_3.	a_4.	b_4.	c_4.	α_4.	
−0′·43	−0′·18	0′·47	246° 51′	+0′·23	+0′·13	0′·27	61° 31′	January.
−0·47	−0·36	0·59	232 44	+0·17	+0·21	0·27	38 0	February.
−0·82	−0·91	1·23	222 13	+0·38	+0·35	0·52	46 48	March.
−0·94	−0·98	1·36	223 43	+0·32	+0·20	0·38	57 19	April.
−1·01	−0·44	1·10	246 41	+0·16	+0·04	0·17	76 48	May.
−0·77	−0·38	0·86	243 58	+0·05	+0·02	0·05	70 30	June.
−0·71	−0·58	0·92	230 25	−0·05	+0·19	0·20	−15 11	July.
−1·04	−0·53	1·17	243 14	+0·15	+0·12	0·19	49 42	August.
−1·08	−0·49	1·18	245 45	+0·50	+0·12	0·52	76 51	September.
−0·82	−0·57	1·00	235 9	+0·43	+0·23	0·49	61 17	October.
−0·56	−0·23	0·61	247 46	+0·30	+0·15	0·33	63 45	November.
−0·32	−0·13	0·35	248 12	+0·15	+0·10	0·18	56 24	December.
−0·748	−0·481	—	—	+0·231	+0·155	—	—	Mean of 12 monthly values.
								Inequality for—
—	—	0·889	237 16	—	—	0·278	56 8	Year.
−0·446	−0·225	0·499	243 14	+0·211	+0·146	0·257	55 19	Winter.
−0·915	−0·736	1·174	231 11	+0·407	+0·228	0·466	60 45	Equinox.
−0·883	−0·481	1·006	241 26	+0·076	+0·091	0·119	39 52	Summer.

Expansion of Diurnal Inequality of Inclination (1891 to 1900).

Period, 8 hours.				Period, 6 hours.				
a_3.	b_3.	c_3.	α_3.	a_4.	b_4.	c_4.	α_4.	
−0′·019	+0′·105	0′·107	−10° 3′	−0′·017	−0′·094	0′·095	190° 6′	January.
−0·049	+0·141	0·150	−19 5	+0·007	−0·103	0·103	175 49	February.
−0·028	+0·207	0·209	− 7 48	−0·041	−0·116	0·123	199 31	March.
−0·059	+0·193	0·202	−16 53	−0·028	−0·101	0·105	195 39	April.
+0·113	+0·112	0·159	+45 14	−0·085	−0·018	0·087	258 6	May.
+0·109	+0·117	0·159	+43 3	−0·053	−0·023	0·057	246 15	June.
+0·051	+0·134	0·143	+20 51	−0·021	−0·043	0·047	207 6	July.
+0·148	+0·181	0·233	+39 14	−0·073	−0·069	0·101	226 37	August.
+0·142	+0·241	0·280	+30 33	−0·091	−0·121	0·151	216 50	September.
−0·009	+0·210	0·210	− 2 19	−0·060	−0·120	0·134	206 36	October.
−0·033	+0·114	0·119	−16 5	−0·054	−0·073	0·091	216 26	November.
−0·021	+0·035	0·041	−30 23	−0·007	−0·029	0·030	193 3	December.
+0·0288	+0·1491	—	—	−0·0435	−0·0758	—	—	Mean of 12 monthly values.
								Inequality for—
—	—	0·1518	+10 56	—	—	0·0874	209 51	Year.
−0·0303	+0·0991	0·1036	−17 0	−0·0174	−0·0745	0·0765	193 9	Winter.
+0·0117	+0·2125	0·2129	+ 3 9	−0·0551	−0·1146	0·1271	205 41	Equinox.
+0·1051	+0·1358	0·1718	+37 44	−0·0581	−0·0383	0·0696	236 36	Summer.

TABLE XXII.—Coefficients and Angles in Fourier Series Expansion

	Period, 24 hours.				Period, 12 hours.			
	a_1.	b_1.	c_1.	α_1.	a_2.	b_2.	c_2.	α_2.
January	+ 2·70	+ 1·06	2·90	68° 32′	−3·73	+0·47	3·76	277° 15′
February	+ 4·77	+ 0·37	4·79	85 36	−3·44	0·00	3·44	269 58
March	+ 8·65	− 1·72	8·82	101 14	−4·99	+2·15	5·44	293 17
April	+12·17	− 4·38	12·94	109 48	−6·85	+3·04	7·49	293 56
May	+11·74	− 9·51	15·11	129 1	−4·32	+4·23	6·05	314 23
June	+10·75	−10·26	14·86	133 39	−4·56	+4·18	6·19	312 29
July	+11·31	− 9·70	14·90	130 38	−5·06	+4·39	6·70	310 57
August	+11·72	− 8·67	14·58	126 29	−3·39	+5·10	6·12	326 21
September	+10·77	− 6·55	12·61	121 18	−2·64	+5·14	5·77	332 50
October	+ 9·82	− 1·83	9·99	100 32	−5·27	+2·41	5·80	294 35
November	+ 5·45	− 0·30	5·46	93 11	−5·12	+1·40	5·31	285 15
December	+ 1·43	+ 0·75	1·62	62 18	−3·07	−0·07	3·07	268 38
Mean of 12 monthly values	+ 8·441	− 4·229	—	—	−4·370	+2·702	—	—
Inequality for—								
Year	—	—	9·443	116 37	—	—	5·138	301 44
Winter	+ 3·588	+ 0·469	3·619	82 33	−3·840	+0·448	3·865	276 39
Equinox	+10·353	− 3·619	10·967	109 16	−4·937	+3·184	5·875	302 49
Summer	+11·381	− 9·536	14·849	129 58	−4·334	+4·474	6·230	315 55

TABLE XXIII.—Coefficients and Angles in Fourier Series Expansion

	Period, 24 hours.				Period, 12 hours.			
	a_1.	b_1.	c_1.	α_1.	a_2.	b_2.	c_2.	α_2.
January	+0·80	−2·43	2·56	161° 43′	−1·15	+0·71	1·35	301° 36′
February	+1·53	−2·80	3·19	151 27	−1·99	+0·60	2·08	286 51
March	+4·97	−2·84	5·10	119 43	−4·64	+0·13	4·64	271 36
April	+6·99	−2·71	7·50	111 12	−5·90	−0·62	5·93	264 1
May	+9·12	−3·78	9·87	112 30	−7·69	+0·57	7·71	274 15
June	+6·75	−5·11	8·46	127 8	−6·07	−0·02	6·07	269 46
July	+7·55	−5·89	9·58	127 57	−6·15	−0·51	6·17	265 14
August	+5·96	−3·34	6·83	119 16	−5·89	+0·76	5·94	277 24
September	+5·86	−3·08	6·62	117 44	−4·86	+0·90	4·94	280 32
October	+4·12	−2·50	4·82	121 13	−3·39	+0·13	3·39	272 15
November	+0·96	−2·34	2·53	157 38	−1·50	+1·23	1·94	309 28
December	+1·09	−1·25	1·66	138 54	−1·12	+0·81	1·38	305 42
Mean of 12 monthly values	+4·64	−3·17	—	—	−4·20	+0·39	—	—
Inequality for—								
Year	—	—	5·62	124·21	—	—	4·21	275 19
Winter	+1·10	−2·21	2·46	153·35	−1·44	+0·84	1·67	300 11
Equinox	+5·49	−2·78	6·15	116·53	−4·70	+0·14	4·70	271 40
Summer	+7·34	−4·53	8·63	121·40	−6·45	+0·20	6·45	271 47

of Diurnal Inequality of Horizontal Force (1890 to 1900). Unit = 1γ.

Period, 8 hours.				Period, 6 hours.				
a_3.	b_3.	c_3.	α_3.	a_4.	b_4.	c_4.	α_4.	
+0·62	−1·70	1·81	159° 47′	+0·18	+1·36	1·38	7° 29′	January.
+1·23	−2·00	2·35	148 25	−0·31	+1·39	1·42	−12 32	February.
+1·44	−3·14	3·45	155 25	+0·01	+1·79	1·79	0 15	March.
+1·92	−2·92	3·50	146 38	+0·18	+1·64	1·65	6 6	April.
−0·66	−1·80	1·91	200 3	+1·13	+0·60	1·28	62 3	May.
−1·01	−2·04	2·28	206 20	+0·32	+0·59	0·67	28 10	June.
0·00	−2·20	2·20	180 0	+0·11	+0·55	0·56	11 9	July.
−1·25	−2·91	3·17	203 16	+0·86	+1·20	1·48	35 35	August.
−1·24	−3·77	3·96	198 10	+1·18	+1·96	2·28	31 5	September.
+1·01	−3·10	3·26	162 1	+0·50	+1·74	1·81	16 10	October.
+0·85	−1·91	2·09	155 59	+0·48	+1·14	1·24	22 58	November.
+0·60	−0·85	1·04	144 33	0·00	+0·64	0·64	0 0	December.
+0·293	−2·361	—	—	+0·386	+1·216	—	—	Mean of 12 monthly values.
								Inequality for—
—	—	2·382	172 56	—	—	1·276	17 37	Year.
+0·828	−1·614	1·814	152 51	+0·089	+1·131	1·134	4 30	Winter.
+0·782	−3·230	3·321	166 23	+0·466	+1·781	1·840	14 40	Equinox.
−0·730	−2·238	2·354	198 4	+0·604	+0·736	0·952	39 22	Summer.

of Diurnal Inequality of Vertical Force (1891 to 1900). Unit = 1γ.

Period, 8 hours.				Period, 6 hours.				
a_3.	b_3.	c_3.	α_3.	a_4.	b_4.	c_4.	α_4.	
+0·50	−0·21	0·54	113° 10′	−0·37	+0·09	0·38	283° 20′	January.
+1·27	−0·11	1·27	94 50	−0·42	−0·10	0·43	256 23	February.
+2·64	−0·03	2·64	90 40	−1·10	+0·15	1·12	277 50	March.
+2·92	−0·09	2·92	91 46	−0·94	+0·38	1·01	291 42	April.
+2·78	−0·57	2·84	101 40	−0·73	+0·40	0·83	298 34	May.
+1·56	−0·52	1·65	108 20	−0·43	+0·32	0·53	306 48	June.
+1·94	+0·28	1·96	81 41	−0·45	−0·34	0·57	233 15	July.
+2·57	−0·64	2·65	103 57	−0·51	+0·16	0·53	287 23	August.
+2·29	−0·79	2·42	109 10	−0·60	+0·50	0·78	309 30	September.
+2·10	−0·43	2·14	101 27	−0·95	+0·12	0·96	276 58	October.
+0·91	−0·39	0·99	113 6	−0·73	+0·23	0·77	287 30	November.
+0·62	−0·27	0·67	113 54	−0·13	+0·03	0·13	283 4	December.
+1·84	−0·31	—	—	−0·61	+0·16	—	—	Mean of 12 monthly values.
								Inequality for—
—	—	1·87	99 40	—	—	0·63	284 38	Year.
+0·82	−0·25	0·86	106 37	−0·41	+0·06	0·42	278 35	Winter.
+2·49	−0·34	2·51	97 40	−0·90	+0·28	0·94	287 34	Equinox.
+2·22	−0·36	2·24	99 17	−0·53	+0·13	0·55	284 13	Summer.

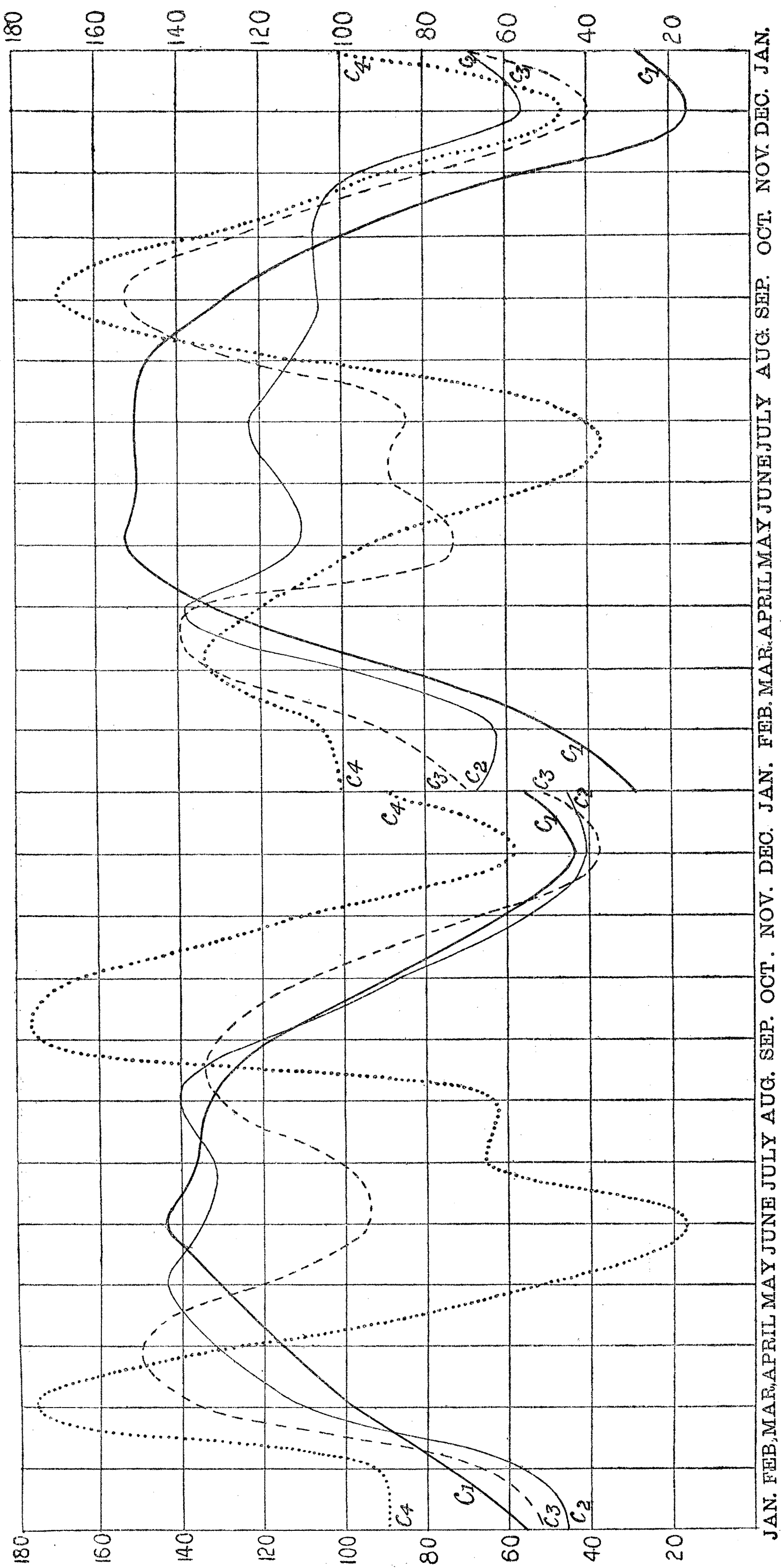
§ 33. The mode of variation of the c coefficients throughout the year is of importance, as its comparison with other phenomena seems one of the most promising methods of investigating the source of diurnal inequality. To facilitate inter-comparisons, I have expressed each monthly value of a "c" as a percentage of the mean value for the 12 months. The results appear in Tables XXIV. and XXV. The tables give also percentage values for the three seasons; these are based on the seasonal diurnal inequalities, 100 representing in their case the arithmetic mean of the values for the three seasons. The mode of annual variation of the c's is also shown graphically in figs. 16 and 17.

It will be readily seen, either from the tables or the curves, that the mode of variation of corresponding c coefficients is fairly similar in all the elements. The coefficient c_1 has an annual variation very similar to that shown by the range of the diurnal inequality (see Table XVIII.). There is the same very pronounced minimum at midwinter, with at least a suggestion of two nearly equal maxima in summer.

The annual variation in c_2 is fairly similar to that in c_1, but the existence of a secondary minimum in summer is usually indicated more clearly. In c_3 the summer minimum is prominent, and in c_4 it is in some cases the chief minimum of the year. The two maxima in c_3 and c_4 are very prominent, and the interval between their occurrence is so much greater than in c_2 or c_1 that they fall approximately at the equinoxes. This question is further dealt with in § 37.

TABLE XXIV.—Variation of Fourier Coefficients throughout the Year. Kew, 11 Years.

	Declination.				Horizontal force.			
	c_1.	c_2.	c_3.	c_4.	c_1.	c_2.	c_3.	c_4.
January	56	46	52	90	29	69	70	102
February	75	56	66	92	48	63	91	105
March	99	110	136	175	89	100	133	133
April	116	131	150	128	131	138	135	122
May	130	144	122	56	153	111	74	95
June	144	135	95	17	150	114	88	50
July	136	132	101	66	151	123	85	41
August	133	141	130	64	148	113	123	110
September	118	119	131	175	128	106	153	169
October	89	88	111	165	101	107	126	134
November	61	56	68	112	55	98	81	92
December	44	42	38	59	16	56	40	47
Winter	59	50	56	92	37	73	73	87
Equinox	105	111	131	166	112	110	133	141
Summer	135	138	113	42	151	117	94	73

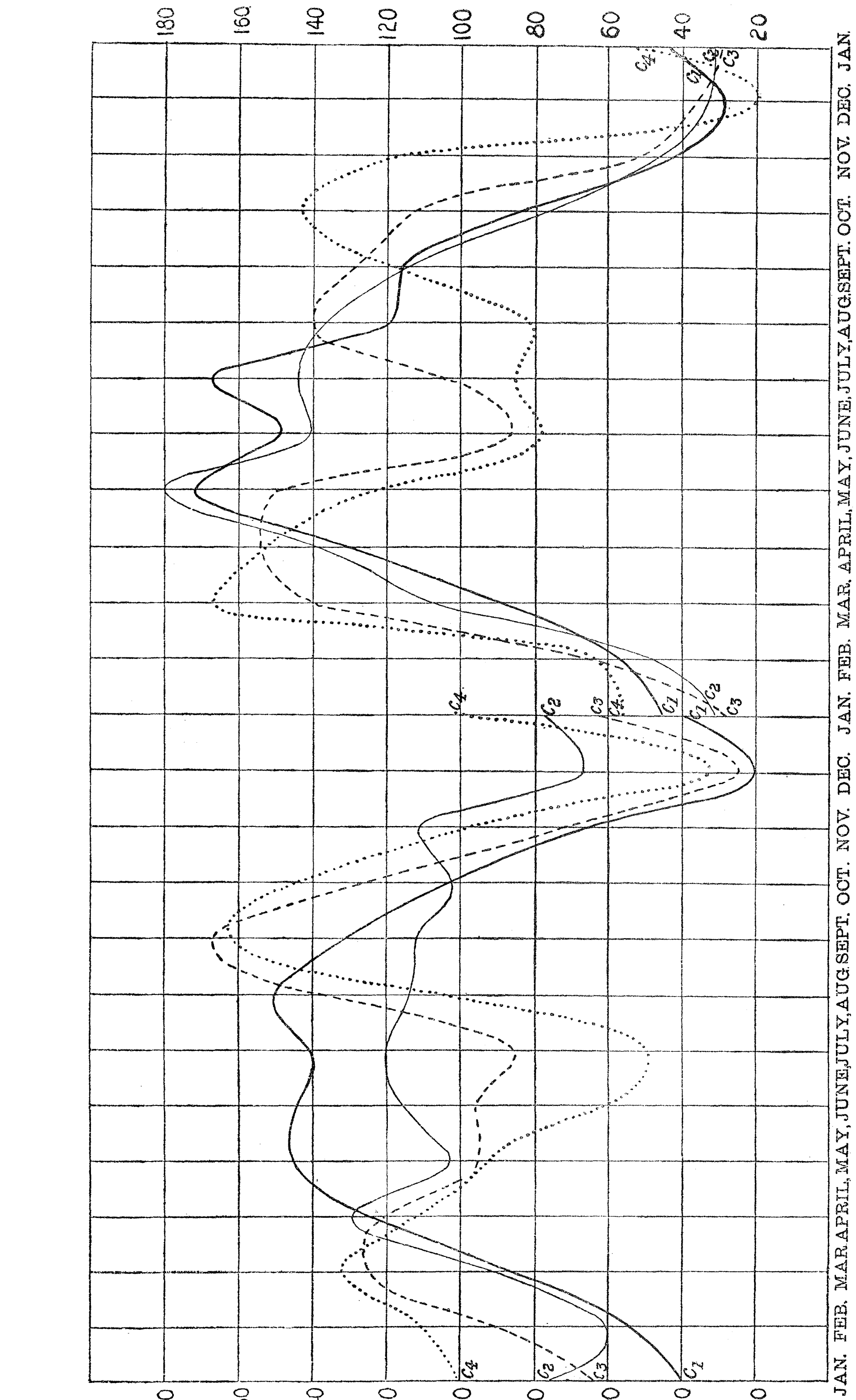


Annual variation of the amplitudes of the principal terms of the Fourier series for the diurnal inequalities (percentage values).

Declination.

Horizontal force.

Fig. 16.



Annual variation of the amplitudes of the principal terms of the Fourier series for the diurnal inequalities (percentage values).

Inclination. Vertical force.

Fig. 17.

TABLE XXV.—Variation of Fourier Coefficients throughout the Year.
Kew, 10 Years.

	Inclination.				Vertical force.			
	c_1.	c_2.	c_3.	c_4.	c_1.	c_2.	c_3.	c_4.
January	40	77	64	101	45	31	29	56
February	53	60	90	110	56	48	67	64
March	86	89	124	132	89	108	140	167
April	125	129	120	112	131	138	154	152
May	145	103	95	93	172	180	150	124
June	144	116	95	61	148	141	87	79
July	140	120	85	50	167	144	104	85
August	150	114	139	107	119	138	140	80
September	128	112	167	162	116	115	128	117
October	103	102	125	143	84	79	113	143
November	66	111	71	97	44	45	52	115
December	20	67	25	32	29	32	36	19
Winter	44	80	64	84	43	39	46	65
Equinox	110	104	131	140	107	110	134	149
Summer	146	116	105	76	150	151	120	86

§ 34. Table XXVI. shows the relative importance of the four leading terms of the Fourier series in the case of the mean diurnal inequality for the year. The importance of the 12-hour term as compared to the 24-hour term appears decidedly greater in D, W, and V, which present remarkably similar features, than in H, N or I, which resemble one another.

Table XXVII. shows how the importance of the higher Fourier terms, as compared to the first, varies with the season of the year. In I and H the relative importance of the higher terms diminishes largely as we pass from winter to equinox, and also conspicuously as we pass from equinox to summer.

In D and V, on the other hand, c_2/c_1 and c_3/c_1 are greatest at the equinox, and c_2/c_1 is decidedly greater in summer than in winter. The summer value of c_4/c_1 is as conspicuously small in D and V as in the other elements, but the equinoctial value is closely similar to the winter value.

TABLE XXVI.—Relations between Fourier Coefficients in mean Diurnal Inequality for the Year.

		c_2/c_1.	c_3/c_1.	c_4/c_1.
Declination	(11 years)	·77	·38	·12
Westerly component	,,	·78	·43	·15
Horizontal force	,,	·54	·25	·14
Northerly component	,,	·58	·19	·09
Inclination	(10 years)	·49	·30	·17
Vertical force	,,	·75	·33	·11
Total force		·65	·25	·08

TABLE XXVII.—Relation between Fourier Coefficients in Diurnal Inequalities for the Seasons (from 11 or 10 years at Kew).

	c_2/c_1.			c_3/c_1.			c_4/c_1.		
	Winter.	Equinox.	Summer.	Winter.	Equinox.	Summer.	Winter.	Equinox.	Summer.
Declination	·65	·81	·78	·36	·47	·32	·18	·19	·04
Inclination	·92	·48	·40	·43	·35	·17	·32	·21	·09
Horizontal force	1·07	·54	·42	·50	·30	·16	·31	·17	·06
Vertical force	·68	·76	·75	·35	·41	·26	·17	·15	·06

§ 35. Taking the 24-hour term, we see in D, I, H and V (see Tables XX. to XXIII.) a fairly regular annual variation of considerable amplitude in the angle α_1. In D and V, α_1 is larger in winter than in summer; in I and H it is the other way about. A larger value of α_1 means an earlier hour for the occurrence of the maximum, an increase of 15° representing an advance of one hour. The angle α_2 increases fairly regularly in passing from winter to summer in the case of D, I and H; but in V the winter value is the largest. The angles α_3 and α_4 show a fairly regular increase from winter to summer in I and H, but in D and V the annual variation seems small.

§ 36. Table XXVIII. gives an analysis of the values of α for the seasonal inequalities when local solar time is used; the corresponding hours of occurrence of the first maximum of the day are shown in Table XXIX. The hours are counted from 0 to 24, with 0 representing true local midnight. The table goes only to tenths of hours, but suffices to show the much greater variability in the time of occurrence of the maximum of the 24-hour wave than in the maxima of the waves of shorter periods. The variability appears greater in I and H than in D or V.

TABLE XXVIII.—Seasonal Values of the Angles in Fourier Coefficient Expansions of Diurnal Inequality when Kew Solar Time used.

Angle.	Declination.			Inclination.			Horizontal force.			Vertical force.		
	Winter.	Equinox.	Summer.	Winter.	Equinox.	Summer.	Winter.	Equinox.	Summer.	Winter.	Equinox.	Summer.
α_1	241° 32′	223° 3′	215° 31′	250° 18′	287° 33′	312° 54′	83° 10′	108° 55′	130° 40′	154° 12′	116° 32′	122° 22′
α_2	31 32	41 43	53 51	92 27	134 13	156 19	277 53	302 7	317 19	301 25	270 58	273 11
α_3	245 5	230 8	243 32	− 15 9	2 6	39 50	154 42	165 20	200 10	108 28	96 37	101 23
α_4	57 47	59 21	42 40	195 37	204 17	239 24	6 58	13 16	42 10	281 3	286 10	287 1

TABLE XXIX.—Time of Occurrence of First Maximum in the Terms of the First 4 Orders in Fourier Expressions for Diurnal Inequality.

Period of Fourier term.	Declination.			Inclination.			Horizontal force.			Vertical force.		
	Winter.	Equinox.	Summer.	Winter.	Equinox.	Summer.	Winter.	Equinox.	Summer.	Winter.	Equinox.	Summer.
	h.	h.	h.	h.	h.	h.	h.	h.	h.	h.	h.	h.
24 hours . .	13·9	15·1	15·6	13·3	10·8	9·1	1·1	22·7	21·3	19·7	22·2	21·8
12 " . .	1·9	1·6	1·2	11·9	10·5	9·8	5·7	4·9	4·4	5·0	6·0	5·9
8 " . .	4·6	4·9	4·6	2·3	2·0	1·1	6·6	6·3	5·6	7·6	7·9	7·7
6 " . .	0·5	0·5	0·8	4·2	4·1	3·5	1·4	1·3	0·8	2·8	2·7	2·7

A reference to Table XI., or fig. 2, shows that the most prominent turning-point in the diurnal inequality for D is the maximum about 1 P.M. (or 13 h.). It will be seen that this comes near the time of occurrence of the single maximum in the 24-hour term, of the second maximum in the 12-hour and 8-hour terms, and of the third maximum in the 6-hour term. A similar coincidence will be found in the case of the most prominent turning-points in I, H, and V.

Variation throughout the Year (Fourier Series).

§ 37. Fourier series may be employed to assist in investigating the variation throughout the year of the diurnal range, or the sum of the 24 hourly differences from the mean, or the values of the c coefficients in the Fourier series representing the diurnal inequalities. A variety of these annual variation series have been calculated and the results appear in Table XXX. In the formulæ t represents an angle increasing at the rate of 30° per *month*, $t = 0$ answering to the *beginning* of January. In the calculations, the results from the monthly inequalities have been treated as corresponding exactly to the middle of the months, and as separated by equal intervals of time. Neither assumption is exactly true. In the selection of the 5 days a month one of the objects kept in view has been that the mean of the 5 days should come near the middle of the month, but in general of course only an approximation is possible. Again, calendar months are unequal in length. Still the mean day of a calendar month seldom differs by more than 24 hours from the position it would occupy if each month were strictly the twelfth of a year, and unless one is dealing with a very long series of years, or with observational data of exceptional accuracy, very little is likely to be gained by replacing calendar months by any theoretically more perfect scheme of days.

In addition to the Fourier series formulæ actually found for the annual variation, Table XXX. gives the ratio existing between:

(i.) P_1, the amplitude of the *annual* term, and M the mean of the 12 monthly values of the quantity considered;

(ii.) P_2, the amplitude of the *semi-annual* term, and M;

(iii.) The amplitudes P_2 and P_1.

The final column in the table gives the mean difference, irrespective of sign, between the observed and calculated values of each quantity for the 12 months, expressed as a percentage of the mean observed value M. In the table, unity represents 1′ in the case of angles and 1γ in the case of force components.

TABLE XXX.—Diurnal Inequalities, Annual Variation in Fourier Series.

		Formulæ.	P_1/M.	P_2/M.	P_2/P_1.	Mean percentage difference between calculated and observed values.
Ranges	D	8·03 + 3·81 sin $(t+275°)$ + 1·22 sin $(2t+273°)$	·47	·15	0·32	4
	I	1·58 + 0·67 sin $(t+264°)$ + 0·16 sin $(2t+269°)$	·42	·10	0·24	5
	H	28·2 + 13·6 sin $(t+269°)$ + 3·0 sin $(2t+261°)$	·48	·11	0·22	3
	V	18·5 + 11·7 sin $(t+282°)$ + 2·2 sin $(2t+242°)$	·63	·12	0·19	5
Sum of 24 differences from mean	D	41·68 + 17·54 sin $(t+274°)$ + 4·97 sin $(2t+291°)$	·42	·12	0·28	2
	I	9·21 + 4·39 sin $(t+264°)$ + 1·15 sin $(2t+263°)$	·48	·12	0·26	4
	H	164·4 + 92·4 sin $(t+269°)$ + 22·5 sin $(2t+256°)$	·56	·14	0·24	4
	V	99·6 + 61·9 sin $(t+281°)$ + 10·3 sin $(2t+252°)$	·62	·10	0·17	7
c_1	D	2·37 + 1·09 sin $(t+274°)$ + 0·17 sin $(2t+304°)$	·45	·07	0·16	2
	I	0·554 + 0·331 sin $(t+266°)$ + 0·079 sin $(2t+266°)$	·60	·14	0·24	5
	H	9·88 + 6·65 sin $(t+269°)$ + 1·47 sin $(2t+261°)$	·67	·15	0·22	3
	V	5·73 + 3·75 sin $(t+279°)$ + 0·38 sin $(2t+222°)$	·65	·07	0·10	9
c_2	D	1·82 + 0·94 sin $(t+276°)$ + 0·25 sin $(2t+271°)$	·52	·13	0·26	5
	I	0·287 + 0·067 sin $(t+254°)$ + 0·032 sin $(2t+222°)$	·24	·11	0·47	9
	H	5·43 + 1·47 sin $(t+270°)$ + 0·78 sin $(2t+241°)$	·27	·14	0·53	8
	V	4·30 + 2·85 sin $(t+281°)$ + 0·62 sin $(2t+246°)$	·66	·14	0·22	7
c_3	D	0·90 + 0·30 sin $(t+281°)$ + 0·32 sin $(2t+274°)$	·33	·35	1·06	6
	I	0·168 + 0·044 sin $(t+251°)$ + 0·072 sin $(2t+293°)$	·27	·43	1·62	7
	H	2·58 + 0·39 sin $(t+241°)$ + 1·05 sin $(2t+290°)$	·15	·41	2·69	8
	V	1·89 + 0·82 sin $(t+284°)$ + 0·80 sin $(2t+276°)$	·43	·42	0·98	9
c_4	D	0·30 + 0·07 sin $(t+115°)$ + 0·18 sin $(2t+279°)$	·24	·59	2·47	15
	I	0·094 + 0·007 sin $(t+146°)$ + 0·043 sin $(2t+286°)$	·08	·46	5·77	12
	H	1·35 + 0·12 sin $(t+126°)$ + 0·64 sin $(2t+285°)$	·09	·47	5·25	12
	V	0·67 + 0·12 sin $(t+303°)$ + 0·26 sin $(2t+259°)$	·18	·39	2·12	16

§ 38. Considering first the last column of Table XXX., we see that the agreement between the observed and calculated monthly values is on the whole best for D and H, and worst for V. This is in accordance with what we would expect from instrumental considerations. Even for V the agreement is very good. Except in the case of c_4, the mean percentage error does not exceed 9, while in the case of the ranges and the sum of the 24 hourly differences from the mean, it does not exceed 7. In the case of c_4 the quantities concerned are so small that an error of 15 per cent. represents but a small fraction of $0'{\cdot}1$ or 1γ, the least quantities actually measured. This shows that annual and semi-annual terms suffice to give a very accurate representation of the annual variation, and that little would be gained by introducing higher terms in the Fourier series.*

Coming next to the ratios, we see that in the case of the ranges, the 24 hourly differences, and c_1, the phenomena presented are fairly similar in all the elements; we have P_1/M varying only from 0·42 to 0·67, and P_2/M varying only from 0·07 to 0·15.

In the case of c_2 the value of P_2/M is nearly the same for all the elements; but, compared to the annual term, the semi-annual term is relatively much more important in I and H than in D and V.

The relative importance of the semi-annual term is much greater in c_3 than in c_1 or c_2; and in c_4 the semi-annual term is much the more important of the two, especially in the case of I and H.

It will be noticed that, speaking generally, the values of P_2/M differ less in the several elements than do the values of P_1/M, and that there is a close resemblance between the phenomena in D and V, and again between the phenomena in I and H.

Coming next to the phase angles, we see that excluding c_4 the different quantities considered show pretty similar results, and that the angles in the annual and semi-annual terms do not differ much. It is probably most instructive to consider not so much the angles themselves, as the results deduced from them for the times of occurrence of the maxima in the several terms. In the case of the annual terms there is only one maximum. Taking, for example, the declination range, we find the angle corresponding to the maximum of the annual term from the equation

$$t + 275^\circ = 450^\circ, \quad \text{or} \quad t = 175^\circ.$$

This answers to 177 days from the beginning of the year, going only to the nearest whole day, and may be taken as answering to June 26. The minimum occurs six months later.

A semi-annual term has of course two maxima separated by six months, with two minima, at three months intervals from the maxima. In the case of the declination range, for instance, the first maximum would answer to

$$2t + 273^\circ = 450^\circ, \quad \text{or} \quad t = 88^\circ{\cdot}5.$$

This may be taken as representing 89 days from the beginning of the year.

* See 'Brit. Assoc. Report' for 1895, p. 223.

The results thus obtained appear in Table XXXI. They should be regarded as affected by an uncertainty of the order of a day through the treatment of the months as of uniform length.

The maximum in the annual term appears in general near midsummer, and would seem somewhat earlier in D and V than in I and H. The first maximum in the semi-annual term appears generally late in March or early in April.

TABLE XXXI.—Annual Variation. Date of Occurrence of First Maximum.

	Annual term.				Semi-annual term.			
	D.	I.	H.	V.	D.	I.	H.	V.
Ranges	June 26	July 8	July 3	June 19	March 30	April 1	April 6	April 16
24 differences	,, 27	,, 8	,, 3	,, 20	,, 21	,, 4	,, 8	,, 10
c_1	,, 27	,, 6	,, 3	,, 22	,, 15	,, 3	,, 6	,, 26
c_2	,, 25	,, 18	,, 2	,, 20	April 1	,, 26	,, 16	,, 13
c_3	,, 20	,, 21	,, 31	,, 17	March 30	March 20	March 22	March 29
c_4	Dec. 6	Nov. 5	Nov. 25	May 29	,, 27	,, 24	,, 24	April 7

"*Variability*" *of the Declination.*

§ 39. In discussing magnetic observations made during the recent "Southern Cross" Antarctic Expedition,* I made a comparison of what I called the "Variability of the Declination" at Cape Adare and at Kew in 1899. I have made a similar investigation into the data at Kew during the remainder of the period 1890 to 1900, partly with a view to seeing whether the phenomenon is of sufficient definiteness and regularity to afford a satisfactory basis for inter-comparison of stations.

It is customary at Kew, during absolute observations, to take two declination readings with the magnet's scale erect, and three with it inverted. These readings show the changes taking place in D during three intervals in which the magnet has remained untouched, the length of an average interval being about 4 minutes. The observations have mostly been made at a time of the day when the regular diurnal change is slow. The same instrument has been in use throughout, the procedure has varied but little, and the same observer has been responsible for the great majority of the observations. Thus the results for different months and years are fairly comparable.

The plan I have adopted has been to sum up all the changes in reading in a year,

* 'Magnetic and Meteorological Observations made by the "Southern Cross" Antarctic Expedition 1898–1900,' p. 24.

or in one month of the 11 years, irrespective of sign, and divide them by the sum of the intervals in minutes between successive readings. The resulting quantity, measured in seconds of arc, I have called the "mean change per minute." The results so found for the several years, and the several months of the year, are given in Table XXXII.

TABLE XXXII.—Variability of the Declination.

Year.	Mean change per minute.	Month.	Mean change per minute.
1890	4″·91	January	3″·39
1891	4·44	February	4·97
1892	5·83	March	4·46
1893	3·91	April	4·21
1894	4·95	May	3·03
1895	4·46	June	3·29
1896	3·24	July	3·68
1897	3·43	August	3·34
1898	2·93	September	4·28
1899	2·95	October	4·27
1900	2·22	November	4·35
		December	3·67
Mean	3·93		

§ 40. If we divide the sum of the changes of reading during the whole eleven years by the sum of the intervals between the successive readings, we find for the "mean change per minute" the value 3″·927, which is in practical agreement with the arithmetic mean of the means for the eleven individual years in Table XXXII.

No observation has been omitted owing to abnormal disturbance, though some 400 to 500 observation days are included in the table. At a polar station no doubt there would be much greater uncertainty on this ground.

The mean values vary on the whole in a fairly regular way from year to year. They have certainly been smaller since 1896 than in previous years; but, as we shall see later, this is only what we should anticipate from sun-spot considerations. The value for 1893 is certainly somewhat conspicuously low compared to the values in 1892 and 1894, while the value for 1890 appears somewhat large. On the whole, however, I think we may conclude that the phenomenon is sufficiently definite to form a satisfactory basis for intercomparison of stations and years. There is a well-marked annual variation proceeding with very fair regularity. There are two minima, near midwinter and midsummer, with maxima near the beginning and end of winter. This is similar to the phenomena observed in declination disturbances,* more

* See MASCART'S 'Magnétisme Terrestre,' Art. 108.

especially at Greenwich. When expressed in a FOURIER series the annual variation shown in Table XXXII. takes the form

$$3''\cdot91 + 0''\cdot377 \sin(t + 88°) + 0''\cdot496 \sin(2t + 292°),$$

where t is measured from the beginning of January exactly as in Table XXX. The semi-annual term is here the larger of the two. Its phase angle differs by only 1° from that obtained in Table XXX. for the corresponding term in the case of the sum of the 24 hourly differences in D. It indicates March 20 as the approximate time for the first maximum.

The annual term has its first maximum about January 2, and its minimum of course six months later; it is thus nearly opposite in phase to the annual term in the expressions for the range in the sum of the 24 hourly differences in D.

Sun-Spot Relations. Introduction.

§ 41. General mention has already been made in § 2, and incidently in other parts of the paper, of relations between magnetic phenomena and sun-spot frequency, and the numerical results obtained by WOLF and WOLFER for sun-spot frequency have been already recorded in Table I. There are other methods of estimating solar disturbance, and it would be of interest to consider likewise data based on these, and to investigate the question as to apparent variations in magnetic phenomena with variation in the position of the spotted areas in the sun's surface. I have thought it better, however, to confine myself wholly on the present occasion to WOLF and WOLFER's numbers, and to cover a limited portion of the ground in a comparatively thorough way, rather than to wander at large over the whole field. My own opinion is that the true inter-relationships of the several phenomena are hardly likely to be reached, and they certainly cannot be *demonstrated*, without a really minute and careful study of the facts.

In treating of sun-spot relations, I shall not adhere strictly to the order followed in the first part of the paper, but shall take first the case of the diurnal periodic changes.

§ 42. It has been established, largely through the work of BALFOUR STEWART and ELLIS, that there is a close connection between epochs of maximum and minimum in sun-spot frequency and in amplitude of diurnal inequality of magnetic elements. In investigating this matter, Mr. ELLIS eliminated the annual variation in the amplitude of magnetic ranges by assigning to any individual month the mean amplitude deduced from twelve consecutive months, of which it was the central month. I have pursued a different method. This consists in expressing the range for any given month as a percentage of the arithmetic mean of the ranges for this particular month of the year throughout the series of years dealt with. This method is obviously less satisfactory when applied to a short number of years than when applied to a period including many sun-spot cycles, a defect from which Mr. ELLIS' method is largely free.

It has, however, the advantage of being equally satisfactory whether the sun-spot and magnetic relationship is or is not variable throughout the year. Mr. ELLIS' method tends to smooth down the magnetic results, and he very properly employs correspondingly smoothed sun-spot numbers, so that the results he obtains are certain *a priori* to present fewer irregularities than those obtained by me. The results I have obtained for D and H appear in Tables XXXIII. and XXXIV. In both cases "range" means the excess of the greatest over the least hourly value in the diurnal inequality derived by combining the five quiet days of each month, the non-cyclic increment being eliminated. The absolute mean value of the range is given for each month of the year, so that the absolute values for individual months can be at once deduced if required.

TABLE XXXIII.—Variation of Declination Range.

(100 × mean for month ÷ mean for that month for 11 years.)

	1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.	1898.	1899.	1900.	Absolute range. Mean for month.
January	92	96	151	140	144	59	126	84	64	59	85	4·39
February	104	77	131	134	146	113	103	82	79	63	68	5·18
March	84	97	108	135	118	113	106	102	77	84	77	9·00
April	93	86	104	125	128	116	101	95	78	96	78	10·68
May	76	103	120	130	118	102	93	87	107	88	77	11·07
June	82	97	126	116	109	138	93	77	82	94	86	10·74
July	89	104	121	125	106	118	96	89	90	79	84	10·48
August	89	93	128	122	124	111	89	89	86	85	84	11·02
September	75	103	116	128	116	102	115	69	90	97	88	9·81
October	89	121	128	120	101	101	98	82	92	79	89	7·63
November	100	138	97	149	124	91	88	74	89	75	75	4·90
December	101	117	160	124	113	109	84	68	78	72	74	3·56
Means for years	89	103	124	129	121	106	99	83	84	81	81	

TABLE XXXIV.—Variation of Horizontal Force Range.

(100 × mean for month ÷ mean of that month for 11 years.)

	1890.	1891.	1892.	1893.	1894.	1895.	1896.	1897.	1898.	1899.	1900.	Absolute range. Mean for month.
January	84	102	152	136	122	93	100	100	71	52	88	$16 \cdot 3\gamma$
February	86	90	134	156	140	120	129	68	52	54	72	$17 \cdot 9$
March	66	85	138	148	114	115	109	82	76	73	94	$27 \cdot 8$
April	83	80	104	142	127	142	95	98	74	82	73	$37 \cdot 5$
May	74	95	110	124	135	124	81	85	103	99	70	$39 \cdot 0_5$
June	86	106	103	119	143	134	83	76	83	83	84	$39 \cdot 0$
July	81	107	122	133	131	118	81	99	79	68	81	$39 \cdot 3$
August	81	115	130	123	119	109	99	74	88	87	75	$38 \cdot 8$
September	68	114	108	125	120	89	123	74	96	102	81	$35 \cdot 7$
October	83	127	116	133	114	97	99	85	86	71	90	$29 \cdot 3$
November	58	139	98	151	162	92	86	70	89	92	62	$20 \cdot 9$
December	106	124	126	162	140	148	61	86	38	58	52	$11 \cdot 9_5$
Means for years	80	107	120	138	131	115	95	83	78	77	77	

§ 43. Before passing to the immediate question of the sun-spot connection, I would call attention to the fact that the values assigned to the ranges in Tables XXXIII. and XXXIV. are without exception greater than those already given in Tables XI. and XIII., and that the differences between the two sets of results are more conspicuous in winter than in summer. This is an exceedingly good illustration of the principle already discussed in § 21. A range in Table XXXIII. is the arithmetic mean of 11 ranges, each based on a combination of only 5 days' results; whereas in Table XI. a range is based on the combination of 55 days' results. The range in Table XXXIII. thus exceeds that in Table XI., the excess being most noticeable at those seasons of the year when the hours of occurrence of maxima and minima are most variable, or when irregular disturbances are largest and most numerous.

§ 44. Referring to Table I. we see that August, 1893, was the month in which sun-spot frequency was greatest. The largest percentage in Table XXXIII. appears in December, 1892, while in Table XXXIV. the largest percentage answers to December, 1893, and November, 1894. The 12 consecutive months for which the sun-spot frequency was largest extended from August, 1893, to July, 1894. The 12 months for which the sum of the percentages is largest run from December, 1892, to November, 1893, in the case of D, and from January to December, 1893, in the case of H. Of calendar years 1893 is that having the largest mean value in all three tables.

The years 1892 to 1895 stand out as years of largest magnetic ranges, but their superiority is not quite so prominent as in the case of sun-spot frequencies; 1890 shows a distinctly smaller sun-spot frequency than any of the 10 following years, but the mean magnetic ranges in 1890 appear slightly greater than in 1899 and 1900. In these two latter years, somewhat curiously, the mean percentages are alike in the case of both D and H. The rise to the maximum is more sudden for magnetic ranges, as well as for sun-spot frequency, than the subsequent decline. This phenomenon has been remarked on by Mr. ELLIS and others.

It will be observed that the percentages in Table XXXIV. show a wider range than in Table XXXIII. The full significance of this will appear presently.

Curves illustrative of Sun-Spot Relations.

§ 45. In the tables devoted to the diurnal inequalities of D, H, N and W the mean inequality for the year is shown not merely for the combined 11 years, but also for two groups of years, viz., 1890, 1899 and 1900, representing small sun-spot frequency, and 1892 to 1895 representing large sun-spot frequency. The great excess of the ranges and the sums of the 24 hourly differences in the second group as compared to the first is conspicuous, but at first sight there is no appreciable difference in the nature of the inequalities shown in the two cases. Fairly definite though minor differences do however exist. If the amplitude of the diurnal inequality alone varied, then curves representing the inequalities for the two groups of years in the ordinary way should transform into one another by merely changing the scale of ordinates, and the same should be true of the difference curve, whose ordinates represent the excess of the ordinates of the sun-spot maximum curve over those of the sun-spot minimum curve. A glance at the curves for D and H, in figs. 18 and 19, shows that this is not the case. The difference curves, due allowance being made for their smaller amplitude, are very decidedly flatter topped than the curves for either group of years. The same phenomenon would be equally shown by N or W curves.

§ 46. Perhaps an even better way of showing the difference in the diurnal inequalities to the eye is to employ vector diagrams. This has been done in figs. 20 and 21. In fig. 20 the mean vector diagram for the whole year has been drawn from the same origin for the two groups of years, while fig. 21 represents a difference vector diagram whose radius vector is the difference between corresponding radii vectors in the two curves of fig. 20. The scale in fig. 21, it should be noticed, is double that in fig. 20. The mean magnetic meridians for the two groups of years are not absolutely identical, but could hardly be shown apart on the scale of the figures.

§ 47. If the sole difference between the inequalities in years of sun-spot maximum and sun-spot minimum were one of amplitude, then the points answering to the same hour

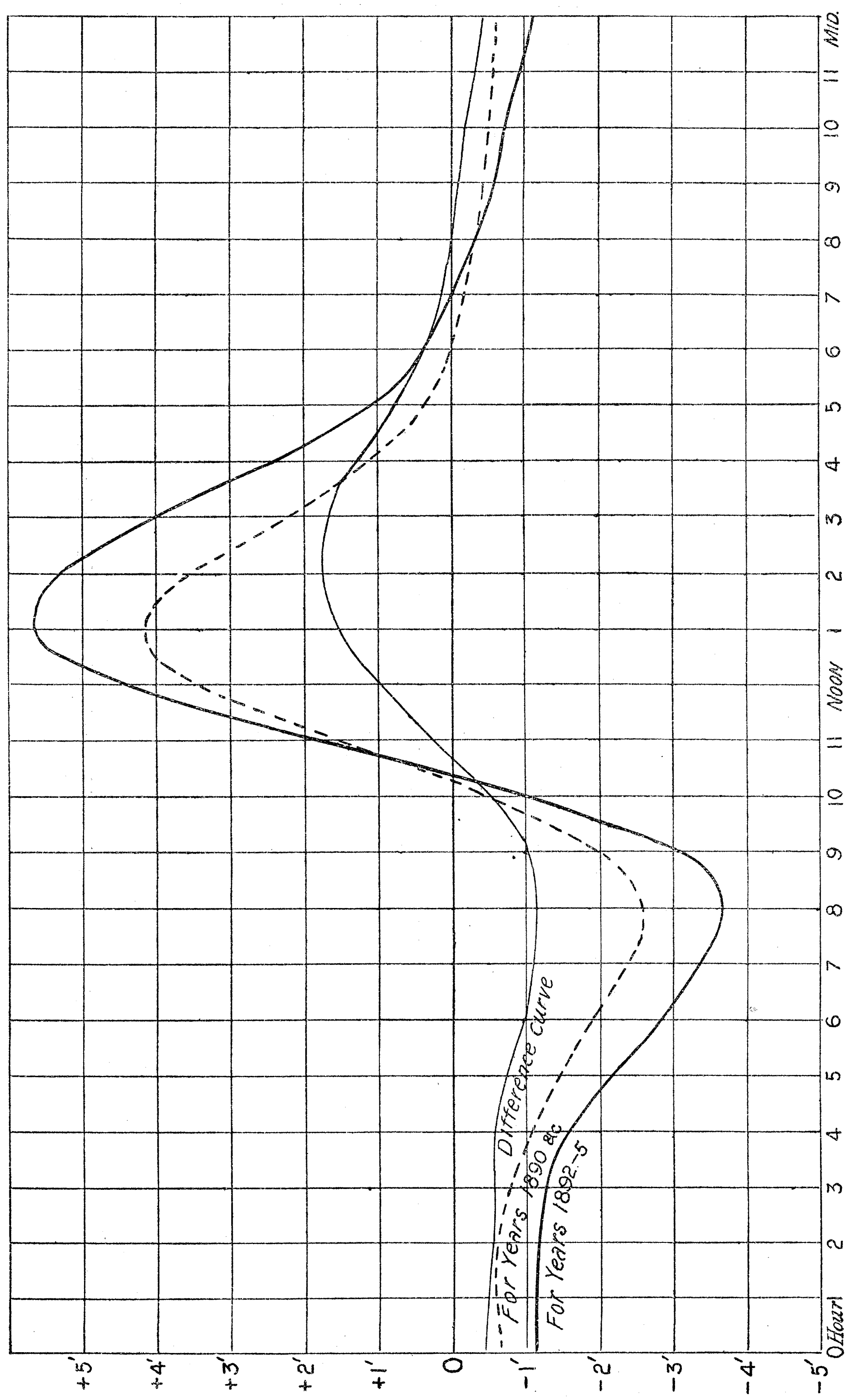


Fig. 18. Mean diurnal inequality of declination for the year.

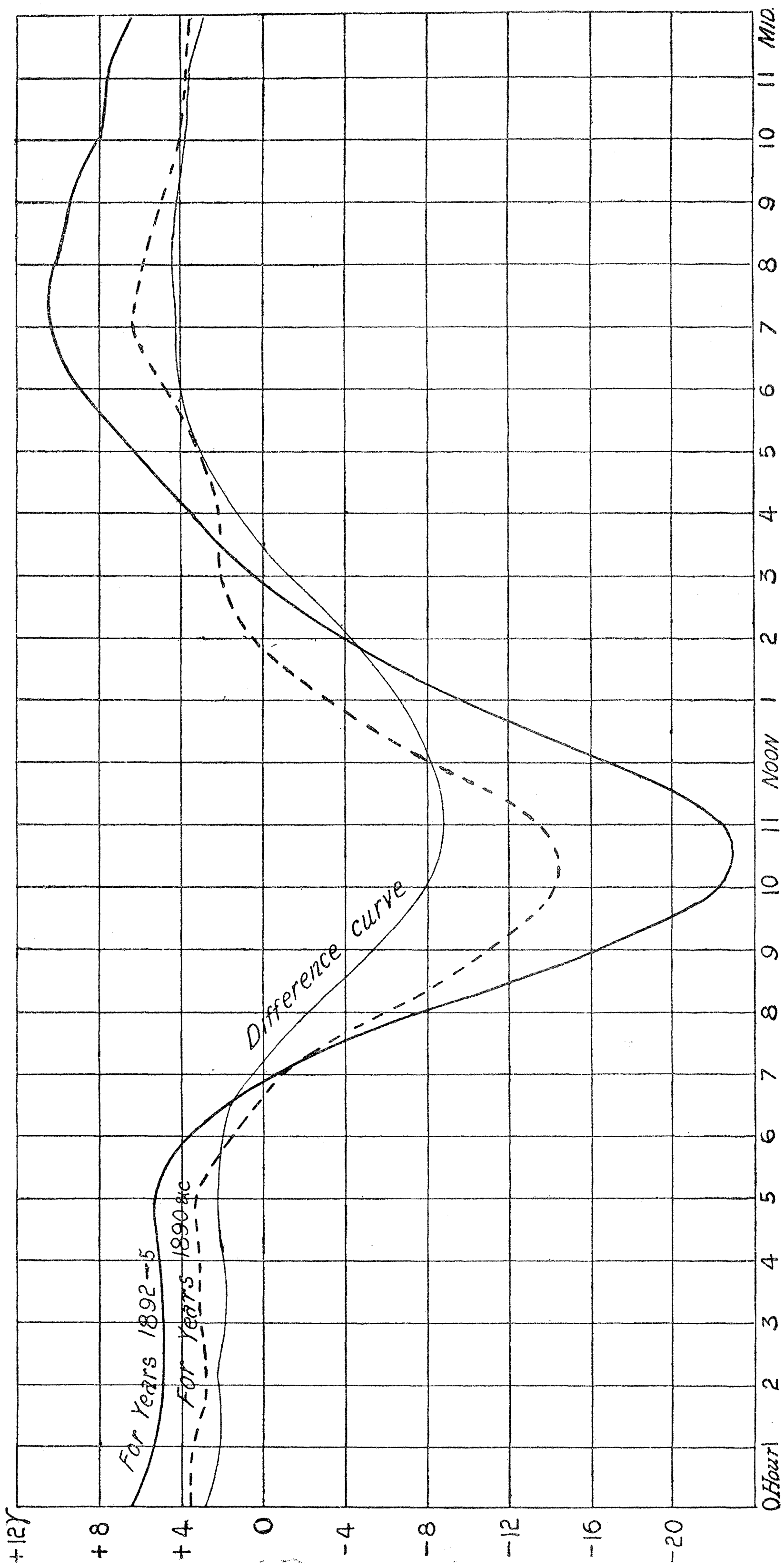
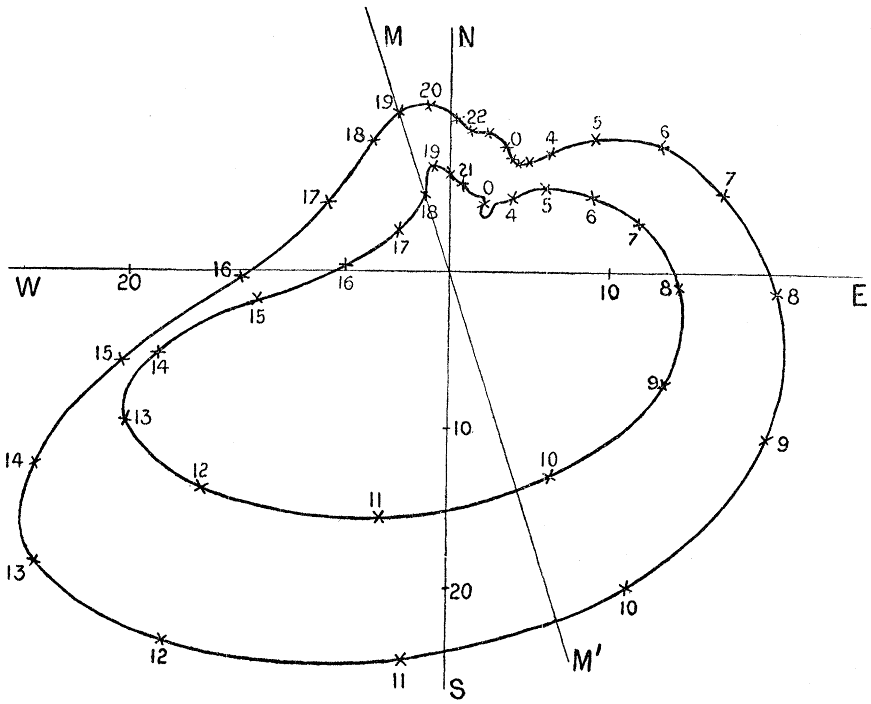


Fig. 19. Mean diurnal inequality of horizontal force for the year.



Inner curve 1890, 1899, 1900. Fig. 20. Outer curve 1892 to 1895.

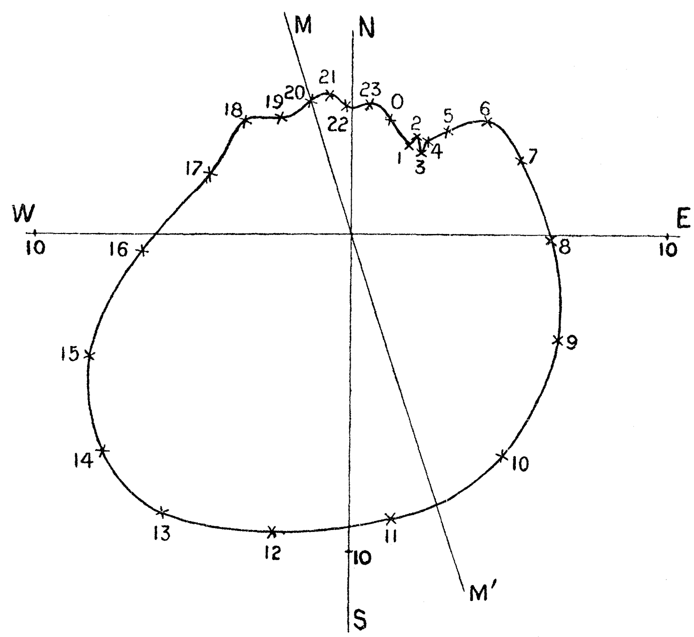


Fig. 21. Difference curve.

in the two curves of fig. 20 should lie on the same radius vector, and the curves should be similar and similarly situated; also the curve in fig. 21 should be similar and similarly situated to the curves in fig. 20. This is obviously not the case. At 9 A.M. corresponding points in the two curves in fig. 20 lie very nearly on the same radius vector, but thereafter the radius vector of the inner curve (sun-spot minimum) forges ahead, and continues ahead until the early morning. The points answering to 1 P.M. and 6 P.M. on the sun-spot minimum curve lie almost exactly on the same radii as the points answering to 2 P.M. and 7 P.M. on the sun-spot maximum curve.

The difference between the shape of the curves in figs. 20 and 21 is conspicuous; the latter curve is much more nearly symmetrical with respect to the geographical meridian than the former.

Sun-Spot Relations apparent in Fourier Coefficients.

§ 48. To push the comparison further, I had Fourier coefficients calculated for the monthly inequalities of D and H from the two groups of years 1890, 1899, 1900 and 1892 to 1895. Corresponding results for the last group of years were also calculated for Greenwich, making use of the values of the "a" and "b" coefficients in the Greenwich annual publications. This gave results more strictly parallel to the Kew results than if one had employed the annual Greenwich values for the "c" coefficients. To facilitate intercomparison of the different cases, I have expressed the monthly values of the c's as percentages of their mean. It is these percentage values that form the main part of Tables XXXV. and XXXVI. The absolute values for each month can however be obtained at once by combining the percentage with the absolute mean value given at the foot of the tables. The data for the 11 years have already appeared in Table XXIV., but it was desirable to show them in juxtaposition to the others. Besides the monthly data, the tables give the values of the c's for the three seasonal inequalities, expressed in percentages of their mean, and likewise the absolute values of the c's in the mean inequalities for the year.—(See pp. 408–411.)

There are of course irregularities in the monthly values, but there is an unmistakable general tendency for the percentage values to rise in winter and fall in summer as we pass from sun-spot minimum years through average years to sun-spot maximum years at Kew.

In the case of D the Greenwich data stand on the far side, so to speak, of the Kew sun-spot maximum data. The same is true for c_3 in H, but in the case of the other three coefficients in H the Greenwich data approach most closely to the average year results for Kew. It would be of interest to know how much of the differences between Kew and Greenwich is due to difference in geographical position, and how much is due to the fact that the Kew data are from quiet days only, while the Greenwich data are from all days.

Tables XXXVII. and XXXVIII. are derived from the same data as Tables XXXV. and XXXVI., but they are intended to show the influence of sun-spot frequency on the relative importance of the *different* Fourier terms.

Table XXXVII. deals only with the mean diurnal inequality for the whole year but gives data for W and N as well as for D and H.

Table XXXVIII. gives seasonal values of the ratios, but for D and H only.—(See p. 412.)

TABLE XXXV.—Annual Variation of Fourier Coefficients in

	c_1.				c_2.			
	Kew.			Greenwich.	Kew.			Greenwich.
	Sun-spot minimum.	11 years.	Sun-spot maximum.	Sun-spot maximum.	Sun-spot minimum.	11 years.	Sun-spot maximum.	Sun-spot maximum.
January . .	53	56	62	63	43	46	47	51
February . .	64	75	84	92	49	56	59	64
March . . .	97	99	96	104	105	110	112	114
April . . .	126	116	115	111	140	131	128	144
May . . .	127	130	129	124	141	144	139	131
June . . .	152	144	144	131	144	135	136	133
July . . .	143	136	131	128	129	132	130	127
August . .	136	133	128	114	143	141	144	127
September .	118	118	113	109	118	119	117	108
October . .	87	89	90	94	88	88	84	91
November .	53	61	58	68	58	56	58	65
December .	44	44	49	62	42	42	44	47
Winter . .	54	59	63	72	48	50	52	57
Equinox . .	107	105	103	104	112	111	110	113
Summer . .	139	135	133	124	139	138	138	130
Absolute values of coefficients in mean diurnal inequality for the year	1·801	2·322	2·858	3·204	1·541	1·789	2·106	2·007
Mean of 12 monthly values	1·859	2·371	2·905	3·313	1·567	1·818	2·146	2·063

the Diurnal Inequalities for Different Groups of Years in D.

c_3.				c_4.				
Kew.			Greenwich.	Kew.			Greenwich.	
Sun-spot minimum.	11 years.	Sun-spot maximum.	Sun-spot maximum.	Sun-spot minimum.	11 years.	Sun-spot maximum.	Sun-spot maximum.	
43	52	50	43	79	90	91	99	January
57	66	72	85	80	92	117	113	February
135	136	137	142	169	175	191	155	March
148	150	156	157	128	128	139	145	April
116	122	117	109	78	56	49	49	May
91	95	94	92	23	17	11	19	June
103	101	100	99	93	66	45	40	July
141	130	134	138	64	64	77	66	August
140	131	128	117	139	175	114	102	September
106	111	106	110	161	165	168	179	October
79	68	67	62	116	112	127	153	November
41	38	39	46	68	59	70	81	December
55	56	57	59	89	92	105	119	Winter
132	131	131	131	158	166	157	153	Equinox
113	113	112	110	53	42	38	28	Summer
0·752	0·889	1·013	0·867	0·281	0·278	0·287	0·260	Absolute values of coefficients in mean diurnal inequality for the year
0·809	0·902	1·026	0·878	0·303	0·296	0·300	0·286	Mean of 12 monthly values

TABLE XXXVI.—Annual Variation of Fourier Coefficients in

	c_1.				c_2.			
	Kew.			Greenwich.	Kew.			Greenwich.
	Sun-spot minimum.	11 years.	Sun-spot maximum.	Sun-spot maximum.	Sun-spot minimum.	11 years.	Sun-spot maximum.	Sun-spot maximum.
January . .	20	29	37	34	59	69	61	63
February . .	40	48	58	44	61	63	73	69
March . . .	81	89	95	85	102	100	102	102
April . . .	124	131	136	136	139	138	139	132
May . . .	162	153	145	141	85	111	124	117
June . . .	165	150	147	164	121	114	116	133
July . .	154	151	148	160	112	123	124	126
August . .	154	148	140	151	118	113	116	113
September .	137	128	114	106	128	106	85	92
October . .	110	101	89	98	132	107	93	113
November .	41	55	64	53	96	98	99	79
December . .	13	16	29	28	48	56	67	61
Winter . .	24	37	47	39	67	73	76	69
Equinox . .	114	112	108	106	124	110	103	108
Summer . .	162	151	145	155	109	117	121	123
	γ.	γ.	γ.	γ.	γ.	γ.	γ.	γ.
Absolute values of coefficients in mean diurnal inequality for the year	6·95	9·44	12·25	13·72	4·02	5·14	6·62	6·91
Mean of 12 monthly values	7·40	9·88	12·82	14·45	4·32	5·43	6·98	7·19

the Diurnal Inequalities for Different Groups of Years in H.

c_3.				c_4.				
Kew.			Greenwich.	Kew.			Greenwich.	
Sun-spot minimum.	11 years.	Sun-spot maximum.	Sun-spot maximum.	Sun-spot minimum.	11 years.	Sun-spot maximum.	Sun-spot maximum.	
84	70	56	81	95	102	111	107	January.
72	91	88	115	78	105	136	76	February.
132	133	137	148	132	133	125	128	March.
125	135	151	130	118	122	127	148	April.
101	74	83	49	144	95	100	106	May.
76	88	96	70	96	50	47	78	June.
115	85	70	70	51	41	40	62	July.
150	123	115	121	111	110	99	67	August.
128	153	151	125	148	169	158	158	September.
108	126	124	137	117	134	133	146	October.
71	81	81	85	83	92	86	73	November.
37	40	48	71	27	47	39	50	December.
67	73	71	90	72	87	97	78	Winter.
121	133	137	132	129	141	139	145	Equinox.
111	94	92	78	99	73	64	77	Summer.
γ.	γ.	γ.	γ.	γ.	γ.	γ.	γ.	
2·14	2·38	2·81	2·93	1·31	1·28	1·35	1·36	Absolute values of coefficients in mean diurnal inequality for the year.
2·43	2·58	3·04	3·21	1·38	1·35	1·47	1·47	Mean of 12 monthly values.

TABLE XXXVII.—Ratios of Fourier Coefficients in Mean Diurnal Inequality for the Year.

	c_2/c_1.				c_3/c_1.				c_4/c_1.			
	D.	W.	H.	N.	D.	W.	H.	N.	D.	W.	H.	N.
Kew sun-spot minimum . .	·86	·88	·58	·61	·44	·49	·31	·23	·16	·20	·19	·12
„ 11 years	·77	·78	·54	·58	·38	·43	·25	·19	·12	·15	·14	·09
„ sun-spot maximum . .	·74	·74	·54	·58	·35	·40	·23	·19	·10	·13	·11	·07
Greenwich sun-spot maximum	·63	—	·50	—	·27	—	·21	—	·08	—	·10	—

TABLE XXXVIII.—Ratios of Fourier Coefficients in Mean Diurnal Inequalities for the Three Seasons.

	c_2/c_1.			c_3/c_1.			c_4/c_1.		
	Winter.	Equinox.	Summer.	Winter.	Equinox.	Summer.	Winter.	Equinox.	Summer.
Declination—									
Kew sun-spot minimum . . .	·76	·88	·84	·44	·53	·35	·25	·23	·06
„ 11 years	·65	·82	·78	·36	·47	·32	·18	·19	·04
„ sun-spot maximum . . .	·61	·78	·76	·32	·45	·30	·17	·15	·03
Greenwich sun-spot maximum .	·49	·68	·65	·22	·33	·23	·13	·12	·02
Horizontal force—									
Kew sun-spot minimum . . .	1·58	·63	·38	·89	·34	·22	·54	·21	·11
„ 11 years	1·07	·54	·42	·50	·30	·16	·31	·17	·06
„ sun-spot maximum . . .	·88	·51	·45	·34	·29	·14	·23	·14	·05
Greenwich sun-spot maximum .	·87	·51	·39	·50	·27	·11	·20	·14	·05

Table XXXVII. shows a steady decline in the relative importance of the higher terms of the Fourier series as the sun-spot frequency increases, and this decline is accentuated in the Greenwich data. Table XXXVIII. shows that the decline apparent in the case of the mean inequality for the year is shared in general by the seasonal inequalities.

§ 49. The angles in the Fourier series were also calculated for each month for the several groups of years. It will suffice, however, to record the results obtained in the case of the inequalities for the three seasons and the year in D and H, adding the values for the year in W and N. These appear in Table XXXIX. The angles all refer to G.M.T., but the differences between the values at the same season of the year would, of course, be the same if local solar time were used.

In the case of the mean inequalities for the year, with the exception of the 6-hour term in H and N, we see a decided decrease of the angle in passing from years of sun-spot minimum, through average years, to years of sun-spot maximum. This means, as already explained, a later hour of occurrence of the maximum in the Fourier series' term. In the case of α_1 the excess of the angle for the sun-spot minimum over the sun-spot maximum years varies from 2° 47′—answering to about 11 minutes in time—in N, to 4° 46′—or about 19 minutes in time—in H; the average difference in time for the four elements, D, W, H, and N, is about 15½ minutes. The corresponding average differences in time in α_2 and α_3 are about 18 minutes and 14 minutes respectively. In α_4 the algebraic mean of the differences for the four elements almost exactly vanishes.

In D in α_1, α_2, and α_3, the difference in angle between sun-spot minimum and sun-spot maximum is conspicuously greatest in winter, and is least in summer. In H the difference in α_1 in winter is opposite in sign to what it is at the other seasons. With the exceptions of α_4 in D, and α_3 in H, the differences between the angles for sun-spot minimum and sun-spot maximum are much the same for the equinox as for the whole year.

TABLE XXXIX.—Angles in Fourier Series for Diurnal Inequalities.

		Declination.				W.	Horizontal Force.				N.
		Winter.	Equinox.	Summer.	Year.	Year.	Winter.	Equinox.	Summer.	Year.	Year.
α_1	Sun-spot minimum	248° 49′	224° 51′	214° 30′	224° 14′	211° 10′	73° 50′	111° 14′	130° 50′	119° 7′	98° 42′
	11 years	240 55	223 24	214 49	222 57	209 9	82 33	109 16	129 58	116 37	97 18
	Sun-spot maximum	235 36	221 27	213 47	221 1	206 28	85 14	106 52	128 57	114 21	95 55
	Sun-spot minimum less sun-spot maximum	**+ 13 13**	**+ 3 24**	**+ 0 43**	**+ 3 13**	**+ 4 42**	**− 11 24**	**+ 4 22**	**+ 1 53**	**+ 4 46**	**+ 2 47**
α_2	Sun-spot minimum	40 25	46 24	55 13	49 33	40 45	289 35	309 59	326 59	311 37	281 37
	11 years	30 18	42 25	52 27	45 3	35 22	276 39	302 49	315 55	301 44	275 24
	Sun-spot maximum	26 3	40 59	50 50	42 57	32 20	272 56	298 37	312 2	297 40	273 39
	Sun-spot minimum less sun-spot maximum	**+ 14 22**	**+ 5 25**	**+ 4 23**	**+ 6 36**	**+ 8 25**	**+ 16 39**	**+ 11 22**	**+ 14 57**	**+ 13 57**	**+ 7 58**
α_3	Sun-spot minimum	255 10	233 51	244 29	241 45	234 23	158 25	165 18	211 15	180 27	143 13
	11 years	243 14	231 11	241 26	237 16	229 43	152 51	166 23	198 4	172 56	135 32
	Sun-spot maximum	236 7	229 5	240 29	234 41	226 32	145 12	164 21	184 8	165 58	130 36
	Sun-spot minimum less sun-spot maximum	**+ 19 3**	**+ 4 46**	**+ 4 0**	**+ 7 4**	**+ 7 51**	**+ 13 13**	**+ 0 57**	**+ 27 7**	**+ 14 29**	**+ 12 37**
α_4	Sun-spot minimum	64 22	59 25	51 45	59 25	50 27	7 14	13 24	24 14	15 29	− 2 54
	11 years	55 19	60 45	39 52	56 8	48 16	4 30	14 40	39 22	17 37	0 14
	Sun-spot maximum	49 25	57 17	37 55	52 13	45 25	5 46	17 58	46 37	20 1	5 3
	Sun-spot minimum less sun-spot maximum	**+ 14 57**	**+ 2 8**	**+ 13 50**	**+ 7 12**	**+ 5 2**	**+ 1 28**	**− 4 34**	**− 22 23**	**− 4 32**	**− 7 57**

Sun-Spot Formula; Numerical Relationships.

§ 50. The previous methods of investigation all point to the conclusion that the relationship between sun-spot frequency and magnetic phenomena varies considerably with the season of the year. To give greater definiteness to the results, I have assumed provisionally a linear relationship*

$$R = a + bS \equiv a \{1 + (b/a) S\}$$

between the range R of a magnetic element and the sun-spot frequency S, with "a" and "b" constants for any one month of the year. I have applied the formula to quantities other than ranges, *e.g.*, to the sum of the 24 hourly differences in the diurnal inequalities, and to the Fourier coefficients of type c. Various methods have also been adopted for calculating a and b.

The first application was to the range data already employed in calculating Tables XXXIII. and XXXIV., and to the corresponding data for I and V. In this case the constants were determined from the 11 values of the range answering to the same month of the year, in the different years, by least squares. The results thus found for a and b and the corresponding values of b/a are given in Table XL.

The values of b and b/a are multiplied by 10^4 so as to avoid decimals. The unit in a is $1'$ in the case of D and I, and 1γ in the case of H and V.

In addition to the values for the individual months of the year, means are also given for the whole year and for the three seasons. It should be noticed, however, that in this case the "a" or "b" assigned to a season (or to the year) is not itself deduced from a seasonal diurnal inequality, but is simply the arithmetic mean of the a's or b's for the individual months of the season (or year); also the value of b/a assigned to a season (or year) is derived from the mean "b" and the mean "a" for the season (or year).

The value of "a" represents of course the value of the range—or other quantity dealt with—which corresponds to a total absence of sun-spots. b is measured in the same units as a. S is simply a number, its values for individual months of the period dealt with being given in Table I.; consequently b/a is also a number, and so independent of the particular units employed in the measurement of force or of angles.

* See "Preliminary Note" for historical references.

TABLE XL.—Values of Constants in Sun-Spot Formula Calculated by Least Squares.

Ranges.

	Declination.			Inclination.			Horizontal force.			Vertical force.		
	a.	$b \times 10^4$.	$(b/a) \times 10^4$.	a.	$b \times 10^4$.	$(b/a) \times 10^4$.	a.	$b \times 10^3$.	$(b/a) \times 10^4$.	a.	$b \times 10^3$.	$(b/a) \times 10^4$.
January	3·16	307	97	0·81	64	80	11·6	118	101	5·3	72	137
February	3·55	383	108	0·62	116	186	10·5	172	164	8·1	75	93
March	7·13	526	74	1·05	145	138	19·3	238	124	18·8	43	23
April	8·56	508	59	1·23	201	164	25·7	284	111	24·5	28	11
May	9·34	401	43	1·58	139	88	29·6	219	74	24·6	153	62
June	8·45	504	60	1·47	161	110	32·6	142	44	23·6	83	35
July	8·75	387	44	1·52	150	98	28·6	238	83	24·7	94	38
August	9·11	421	46	1·89	101	53	31·6	160	51	21·2	44	21
September . . .	7·48	517	69	1·48	137	92	27·0	192	71	14·3	125	88
October	6·13	361	59	1·28	106	83	22·2	171	77	12·3	91	74
November . . .	3·60	369	102	0·80	144	180	13·0	225	173	8·5	38	44
December . . .	2·62	234	89	0·30	95	316	6·7	131	193	6·1	15	24
Mean of monthly values for—												
Year	6·49	410	63	1·17	130	111	21·5	191	89	16·0	72	45
Winter . . .	3·23	323	100	0·63	105	166	10·5	161	145	7·0	50	71
Equinox . .	7·32	478	65	1·26	147	117	23·5	221	94	17·5	72	41
Summer . .	8·91	428	48	1·61	137	85	30·6	190	62	23·5	94	39

§ 51. The variation of "*a*" throughout the year in Table XL. is fairly similar to what we have already observed in the case of the ranges from 11 year means. There is a conspicuous minimum in winter, usually in December, and at least a trace of a second minimum near midsummer. The fluctuations of b and of b/a from month to month are somewhat irregular, and are doubtless in part purely accidental. It would probably require a long series of years, including several sun-spot cycles, to give a smooth annual variation. The general features are however clear enough, and may be easily grasped from a comparison of the mean values assigned to the three seasons. In all four elements b is decidedly least, but b/a decidedly greatest, in winter. A smaller value of b implies a smaller absolute increase to the value of an element for a given increase of sun-spot frequency. A larger value of b/a means a larger percentage change in the element for a given change in sun-spot frequency.

The value of b/a appears less in summer than at the equinox in all four elements, and this is even true, though much less conspicuously, of the value of b itself in all the elements except V.

For such practical purposes as navigation and survey work the value of b is probably the most important thing, but for theoretical work, especially when dealing with data from a single station, the value of b/a will probably prove to possess more significance. If we divide the yearly and seasonal values of $(b/a) \times 10^4$, in Table XL., by 5, 4, 3 and 2 respectively, we get the following results, going to the nearest integer :—

	Year.	Winter.	Equinox.	Summer.
$\frac{1}{5}(b/a)\,10^4$ in I	22	33	23	17
$\frac{1}{4}(b/a)\,10^4$ in H	22	36	23	16
$\frac{1}{3}(b/a)\,10^4$ in D	21	33	22	16
$\frac{1}{2}(b/a)\,10^4$ in V	22	35	21	19
Mean	22	34	22	17

This brings out the remarkable similarity in the seasonal changes of b/a in the different elements, and the closeness between the values of b/a for the equinox and for the year as a whole.

Whilst the sun-spot influence as given by Table XL. is decidedly least for V, the difference from the other elements is less than would appear from the "Preliminary Note." The results given there for V were appreciably influenced by the neglect of the temperature correction in years prior to 1897, already explained in § 19. The omission of the correction diminished the range in V, and brought its values in years prior to 1897—mostly years of large sun-spot frequency—closer to the values in the later years than they ought to have been.

§ 52. Ranges based on only five quiet days are somewhat uncertain, and calculations by least squares are somewhat laborious, I thus investigated a second method of finding values of a and b, which seems to work satisfactorily. It makes use of values deduced from the three groups of years 1892 to 1895, 1890, 1899, 1900, and 1890 to 1900. The method will be most easily grasped by considering a concrete case. For this purpose, let us consider the declination range for January. We have the following data :—

Inequality from 1892 to 1895 . .	Range 4′·97	Sun-spot frequency . .	72·65
,, ,, 1890, 1899, 1900 .	,, 3′·26	,, ,, . .	11·40
Differences	1·71		61·25

Consequent value for $b = 1{\cdot}71 \div 61{\cdot}25 = 279 \times 10^{-4}$.

Inequality from 1890 to 1900 . . Range 4′·07 Sun-spot frequency . . 39·8.

Hence value for $a = 4'{\cdot}07 - 39{\cdot}8 \times 279 \times 10^{-4} = 2'{\cdot}96$.
And value for $b/a = {\cdot}0279 \div 2{\cdot}96 = 94 \times 10^{-4}$.

The values obtained in this way for the ranges and for the sums of the 24 hourly differences in the diurnal inequalities of D and H are given in Tables XLI.

TABLE XLI.—Values of Constants in Sun-Spot Formula, Calculated from Groups of Years.

	Declination.						Horizontal force.					
	Ranges.			Sum of 24 differences.			Ranges.			Sum of 24 differences.		
	a.	$b \times 10^4$.	$(b/a) \times 10^4$.	a.	$b \times 10^3$.	$(b/a) \times 10^4$.	a.	$b \times 10^3$.	$(b/a) \times 10^4$.	a.	$b \times 10^2$.	$(b/a) \times 10^4$.
January	2·96	279	94	15·07	216	144	10·4	123	118	33·2	82	246
February	3·06	398	130	17·43	313	179	9·2	177	192	41·1	113	274
March	6·35	695	109	30·70	378	123	15·5	283	183	75·6	211	279
April	8·84	416	47	41·30	208	50	23·4	295	126	137·0	191	139
May	8·63	530	61	39·82	312	78	28·5	222	78	168·2	154	92
June	8·28	515	62	45·15	258	57	28·8	217	75	164·8	145	88
July	7·90	509	64	40·84	277	68	28·2	256	91	154·5	184	119
August	9·00	443	49	45·52	206	45	30·2	165	55	172·8	120	70
September	7·41	520	70	34·04	330	97	26·4	186	71	147·8	110	75
October	6·05	351	58	28·26	266	94	21·9	128	58	134·1	82	61
November	3·71	296	80	18·58	228	123	12·2	221	181	52·2	158	302
December	2·63	177	67	12·28	153	124	5·2	134	257	21·0	67	320
Mean of monthly values for—												
Year	6·24	427	68	30·75	262	85	20·0	200	100	108·5	135	124
Winter	3·09	287	93	15·84	228	144	9·3	164	177	36·8	105	285
Equinox	7·16	496	69	33·58	296	88	21·8	223	102	123·6	149	120
Summer	8·45	499	59	42·83	263	62	28·9	215	74	165·1	151	91

§ 53. As in Table XL., the yearly and seasonal values of a and b given in Table XLI. represent simply arithmetic means of the values for the individual months, and the values assigned to b/a are obtained by combining the b for the season with the mean a. The units employed are, as in Table XL., 1′ for D, and 1γ for H.

In comparing Tables XL. and XLI., it should be remembered that the ranges in the former are derived from five days in one year, while in the latter they are derived from a group of years supplying at least 15 days. This implies, as already pointed out, that the average range dealt with in Table XL. is larger than the average range dealt with in Table XLI.

It will be observed that the yearly and seasonal values of "a," given in Table XL., are invariably larger than those given in Table XLI., and the same is true of the great majority of individual months. On the other hand the yearly and seasonal values of b, given for D and H, in Table XL., are, with one exception, less than the corresponding values given in Table XLI. A third point of difference is, that the equinoctial values of b for the ranges in Table XLI. have not the pre-eminence compared to the summer values that they show in the corresponding cases in Table XL.

Whilst minor differences exist, the main features in Tables XL. and XLI. are alike. Both show, in winter, a conspicuous minimum in b, but a conspicuous maximum in b/a. Both make the equinoctial values of b/a nearly equal to the mean values for the year. The values found for b and b/a, in individual months, are evidently affected by considerable uncertainties, but the modes of annual variation in the two tables resemble one another more closely than I had anticipated.

It will be noticed that the values found for b/a for the sum of the 24 hourly differences in Table XLI. exceed the corresponding values for the ranges in every single case in H, and in all but two individual months in D. This difference between the two sets of values of b/a is very decidedly larger in winter than in summer. The inference is that sun-spot influence on Terrestrial Magnetism is really underestimated if we confine our attention to the *range* of the diurnal inequalities, and this is especially true of winter.

The phenomena, one need hardly say, absolutely bear out the remarks made in § 45 as to the flat-topped character of the difference curves in figs. 18 and 19.

§ 54. The method on which Table XLI. is based, has also been applied to the "c" Fourier coefficients in the diurnal inequalities. The results thus obtained from the mean annual and seasonal inequalities for the three groups of years are given in Table XLII. Yearly data are given for N and W, as well as for D and H.

For comparison, yearly results are also given for Wilhelmshaven, based on tables of values of the c coefficients for individual years, from 1889 to 1895, published by Dr. BÖRGEN.* I have employed 1892 to 1895 for the group of sun-spot maximum

* 'Beob. aus dem Magn. Obs. zu Wilhelmshaven,' Fünfter Theil, pp. 46 *et seq.*

years, and 1889, 1890 as the group of sun-spot minimum years in dealing with the Wilhelmshaven data. The latter group, it will be noticed, differs from that used for Kew.

The Wilhelmshaven data also differ in being from all days in the year, and in their case one had to employ arithmetic means of the *c*'s from individual years instead of employing values answering to the inequality from the group of years combined. This last difference would inevitably tend to increase the size of the Wilhelmshaven coefficients as compared to the Kew, though probably to but a trifling extent.

As the declination at Wilhelmshaven is some $4\frac{1}{2}^\circ$ less westerly than at Kew, the results for N and W at the two places are presumably less directly comparable than the results for H.

TABLE XLII.—Values of Constants in Sun-Spot Formula, Calculated from Groups of Years. Fourier Coefficients in Expressions for Diurnal Inequality.

		c_1.			c_2.			c_3.			c_4.		
		a.	$b \times 10^4$.	$(b/a) \times 10^4$.	a.	$b \times 10^4$.	$(b/a) \times 10^4$.	a.	$b \times 10^4$.	$(b/a) \times 10^4$.	a.	$b \times 10^4$.	$(b/a) \times 10^4$.
Declination, unit = 1′	Kew winter	0·87	134	155	0·68	59	87	0·41	23	56	0·22	8	38
	,, equinox	1·77	174	99	1·59	103	65	0·98	48	49	0·46	2	3
	,, summer	2·43	170	70	2·03	104	51	0·86	32	37	0·14	− 5	− 38
	,, year	1·65	162	98	1·43	86	61	0·75	34	46	0·27	1	3
Horizontal force, unit = 1γ	Kew winter	1·00	663	660	2·34	386	165	1·52	76	50	0·88	64	72
	,, equinox	7·04	959	136	4·50	334	74	2·51	198	79	1·67	42	25
	,, summer	10·81	903	84	3·96	506	128	2·30	11	5	1·19	− 53	− 44
	,, year	6·06	811	134	3·48	398	115	1·95	103	53	1·25	7	5
	Wilhelmshaven year	7·39	876	119	4·18	527	126	2·47	114	46	1·24	31	25
Northerly component, unit = 1γ	Kew year	7·02	877	125	4·29	461	107	1·57	116	74	0·94	− 6	− 6
	Wilhelmshaven year	9·30	886	95	4·99	518	104	2·41	111	46	1·01	29	28
Westerly component, unit = 1γ	Kew year	8·16	766	94	7·24	392	54	4·15	173	42	1·67	13	8
	Wilhelmshaven year	9·73	785	81	7·25	480	66	3·43	223	65	1·55	31	20

§ 55. Considering first the Kew data in Table XLII. by themselves, we see that the phenomena presented in D and H by c_1—*i.e.*, the 24-hour term—are very similar to those already described in the case of the ranges. Absolutely considered, b is least in winter, and is, if anything, slightly greater at the equinoxes than at midsummer; the value of b/a is conspicuously greatest in winter, and least in summer. The value of b/a for c_2 is also largest in winter, but in the case of H the summer value appears also in excess of the equinoctial. At Kew, the values of b/a for c_2 are with one exception decidedly smaller than the corresponding value for c_1; in like manner the values for c_3 are generally less than those for c_2, and the values for c_4 less than those of c_3. In fact, in summer we have negative values for b/a in c_4 for D, H and N. This may be accidental, as the numerical values appear very small, but we may at least conclude that sun-spot frequency exerts but a trifling influence on the value of c_4 at Kew.

The reduction in the value of b/a as we pass from c_2 to c_3, and from c_3 to c_4, is also well shown at Wilhelmshaven, though not quite so prominently as at Kew. Where the Wilhelmshaven data differ most notably from those at Kew, is in giving a larger value of b/a for c_2 than for c_1, in both H and N.

The values of b/a found for c_1 at Kew appear decidedly larger than those found in Table XLI. for the range, or even as a rule than those found for the sum of the 24 hourly differences.

Comparing the different elements at Kew, we see that in c_1 and c_2 the mean values of b/a for the year are pretty much alike in H and N, and again in D and W, the values appearing slightly larger in H than in N, and very slightly smaller in W than in D. The quantities c_3 and c_4 are themselves so small that conclusions based on the apparent differences in their case in the different elements, would possess an uncertain value.

Comparing Wilhelmshaven with Kew, we see that in the case of c_1 the Wilhelmshaven values are in excess of the Kew for both a and b; but the differences between the corresponding values for b are so small that the Kew values for b/a are decidedly the larger. In the case of c_2 all the figures for Wilhelmshaven are in excess of the corresponding figures for Kew, except the value of b/a in N. In c_3 the Wilhelmshaven values of b/a are the larger in W, but the smaller in H and N. In c_4 the Wilhelmshaven values for b and b/a are decidedly larger than the Kew, but still very small. Whilst differences exist, there is a sufficiently close resemblance between the results at the two places to prove that the phenomena observed at Kew are by no means of an exceptional character.

§ 56. The method of employing three groups of years has also been applied to the ranges and the sums of the 24 hourly differences in the mean diurnal inequalities for the year. The object was partly to obtain a comparison with Parc St. Maur through the intermediary of data published by MOUREAUX.* These data are the values of

* 'Ann. du Bureau Central Météorologique de France,' 1899, "Mémoires," p. B.9.

the ranges in the mean diurnal inequalities for the individual years of the period 1889 to 1899. For sun-spot maximum at Parc St. Maur I took the years 1892 to 1895, and for sun-spot minimum 1889, 1890 and 1899. The value assigned to a group of years was in this case the arithmetic mean of the values for its individual years.

In the case of Kew, the results for I and V were obtained exactly like those for Parc St. Maur, except that 1900 took the place of 1889 and 1890; but in D, H, N and W, I took the values given by the inequalities for the group of years combined, as given in Tables XI., XIII., XV., XVI. The reason for doing this was that individual years' results for N and W did not exist, and it was desirable that these two elements and D and H should be treated exactly alike. The difference in the method of treatment would tend slightly to reduce the values of the ranges in D, H, N and W at Kew as compared to Parc St. Maur.

For facility of comparison, the results for the ranges at the two places are juxtaposed. The angular and force units are respectively 1′ and 1 γ.

TABLE XLIII.—Values of Constants in Sun-Spot Formula, from mean Diurnal Inequalities for Individual Years or for Combinations of Years (Years Grouped).

	Kew.						Parc St. Maur.		
	24 differences.			Ranges.			Ranges.		
	a.	$b \times 10^3$.	$(b/a) \times 10^4$.	a.	$b \times 10^4$.	$(b/a) \times 10^4$.	a.	$b \times 10^4$.	$(b/a) \times 10^4$.
D . . .	30·5	261	86	6·20	407	66	7·27	429	59
W . . .	150·1	1234	82	32·4	1850	57	38·3	2358	62
H . . .	99·2	1304	131	18·3	1896	104	20·4	2170	106
N . . .	117·9	1491	126	19·4	1942	100	22·2	2257	102
I . . .	4·91	75	153	0·89	120	135	1·14	132	116
V . . .	77·8	422	54	14·5	779	54	16·7	1003	60

§ 57. Comparing the Kew results in Table XLIII. with the corresponding results in Table XLI., we find an excellent agreement, especially in the case of D. Comparing the Kew data for the ranges in Table XLIII. with the corresponding data in Table XL., we see that the values found for "a" in Table XLIII. are always slightly the smaller. This is also true of the values of "b" except in the case of V. On the other hand, the values found for b/a in Table XLIII. are larger than those given in Table XL. The differences between the two tables are by no means large, but they emphasise the necessity of bearing in mind the principle explained in § 21. Still confining our attention to the Kew data, we see that, as in the case of c_1 in Table XLII., the values of b/a for D and W come close together, the former being slightly the larger, while the values of b/a for H and N also differ but slightly, and in the same direction in the two cases.

Coming now to the results for Parc St. Maur, we see that the values for "a" are all in excess of the corresponding values for Kew, but the values of b/a at the two places are in general closely alike, the difference being sometimes in one direction, sometimes in the other. The agreement in the values of b/a at the two places is closest in H and N, and least close in I. The fact that at Parc St. Maur V shows a slightly larger value of b/a than D is noteworthy.

On the whole, the similarity in the phenomena shown at Kew and Parc St. Maur is exceedingly satisfactory. The resemblance appears decidedly closer than between Kew and Wilhelmshaven. This is probably partly due to the fact that the declinations at Parc St. Maur and Kew differ only about 2°.

§ 58. Table XLIV. gives a final set of results from the mean diurnal inequalities for the year. They were obtained from the individual inequalities for 11 or 10 years by least squares. The agreement with the results in Table XLIII. is good. This is satisfactory, if only because it shows that it is possible to obtain by means of the much simpler method of Table XLIII. results giving a good approximation to those found by least squares.

TABLE XLIV.—Values of Constants in Sun-Spot Formula from Mean Diurnal Inequalities in Individual Years by Least Squares.

	24 differences.			Ranges.		
	a.	$b \times 10^3$.	$(b/a) \times 10^4$.	a.	$b \times 10^4$.	$(b/a) \times 10^4$.
Declination . . . (unit 1′)	30·5	264	87	6·10	433	71
Inclination . . . " "	4·63	81	175	0·87	125	145
Horizontal force (unit 1γ)	97·8	1358	139	18·1	1942	107
Vertical force . . " "	77·0	439	57	14·3	806	56

§ 59. I have also applied the sun-spot formula to the results for non-cyclic effect in I and H given in Tables VI. and VII., and to the monthly results obtained for the variability of D as explained in § 39. The years were grouped after the fashion explained in § 52.

The results are given in Table XLV. for the whole year, and for the three seasons. Owing to the extreme smallness of the observational quantities, it would require a very long series of years to eliminate accidental sources of error. Thus the seasonal results, more particularly for the non-cyclic effect, are probably affected by considerable uncertainties, especially as regards the values of b/a. In fact, my object in recording the seasonal values was largely to enable the reader to judge for himself what attitude he should adopt towards the mean results for the year.

TABLE XLV.—Values of Constants in Sun-Spot Formula.

	Non-cyclic effect.						Variability of declination in seconds of arc per minute.		
	Horizontal force (unit 1γ).			Inclination (unit $1'$).					
	a.	$b \times 10^4$.	$(b/a) \times 10^4$.	a.	$b \times 10^5$.	$(b/a) \times 10^4$.	a.	$b \times 10^4$.	$(b/a) \times 10^4$.
Winter	1·50	507	338	0·07	470	702	3·00	289	96
Equinox . . .	2·77	217	78	0·22	90	42	3·32	240	72
Summer . . .	2·10	171	81	0·17	124	71	2·49	188	76
Year	2·12	292	138	0·14	251	179	2·93	239	81

§ 60. The conclusions I myself draw from Table XLV. are that non-cyclic effect and variability of declination are both strongly influenced by sun-spot frequency, and that this influence is decidedly most prominent in winter, though hardly I should imagine to the extent indicated by the values found for b/a in the non-cyclic effects.

One exceptional feature in Table XLV. is that "a" appears greatest, and b/a least, at the equinox. This might arise from an over-estimate of "a."

It will be noticed that the mean values for the year of b/a in non-cyclic effect in H and I, and in the variability in D, are remarkably close to the corresponding values found in Table XLIV. for the case of the 24 hourly differences in these respective elements.

When we find two phenomena presenting similar laws of variation, whilst the conditions are artificially varied in a definite manner, we infer at least an intimate relationship between their causes. One is thus tempted to deduce that the source, or sources, of the non-cyclic effect is not absolutely distinct from that of the regular diurnal inequality. If, for instance, the inequality is due to electric currents in the upper atmosphere, one would be disposed to infer that the source of the non-cyclic effect is there also. It seems improbable that two absolutely distinct loci would be so similarly influenced by the changes, whatever they are, that proceed *pari passu* with sun-spot variations.

It is unsafe to base theories on a few numerical coincidences, which after all may be accidental, but I hope it will at least be allowed that non-cyclic effect and declination variability are true manifestations of terrestrial magnetism, and that the phenomena they exhibit at different stations and over different periods of time form rather a promising field of investigation.

Relations with Meteorological Phenomena.

§ 61. Evidence has been adduced by many physicists, including Dr. BALFOUR STEWART and Sir NORMAN LOCKYER, suggestive of connection between sun-spots and various meteorological phenomena, such as rainfall, barometric pressure, &c. It is thus desirable to see what was happening meteorologically at Kew from 1890 to 1900, and, more especially, whether there was or was not any appreciable difference between the meteorological data for the two groups of years 1892 to 1895 and 1890, 1899 and 1900.

Table XLVI. gives particulars of the mean values of the chief meteorological elements at Kew for the whole period, and for the two shorter groups of years. In the case of vapour pressure, 1891 was omitted, as reliable data were wanting for two or three months of the year. The mean daily range of temperature in the table is the mean excess of the maximum over the minimum on individual days.

What impresses one chiefly on studying the table, is the remarkably close agreement between the means for such comparatively short periods of years. When the means for the two shorter periods differ, they are as often as not on the same side of the mean for the 11-year period.

TABLE XLVI.—Mean Values of Meteorological Elements at Kew.

Period.	Mean temperature for the day.	Mean daily range of temperature.	Mean hours of sunshine for year.	Mean vapour pressure.	Mean amount of cloud (Total = 10).	Mean total annual rainfall.	Mean height of barometer.	Average wind velocity, miles per hour.
	° F.	° F.		inch		inches	inches	
1890 to 1900 . .	49·71	13·68	1521	0·287	6·5	22·360	29·978	10·21
1892 to 1895 . .	49·48	13·90	1536	0·283	6·2	23·538	29·964	10·20
1890, 1899, 1900 .	49·87	13·83	1584	0·288	6·5	21·310	29·970	10·20

§ 62. As results from the diurnal inequalities are more strictly parallel to most of the Kew magnetic data, mean diurnal inequalities of temperature and barometric pressure—the two most important meteorological elements—were calculated from the data published by the Meteorological Office in their "Hourly Observations." This was done for groups of sun-spot maximum and sun-spot minimum years. Data for 1900 being not yet available, 1889 was substituted in the sun-spot minimum group. The following were the results :—

RANGES in Mean Diurnal Inequalities for the Year.

Group of years.	Temperature.	Barometric pressure.
	° F.	inch.
1892 to 1895	10·00	·0265
1889, 1890, 1899	9·70	·0267

DIURNAL Inequalities Expressed in Fourier Series.

Group of years.	Temperature, in degrees Fahrenheit.
1892 to 1895	$4^\circ\cdot 91 \sin(t+224^\circ\cdot 6)+0^\circ\cdot 79 \sin(2t+43^\circ\cdot 7)$ $+0^\circ\cdot 14 \sin(3t+17^\circ\cdot 7)+0^\circ\cdot 06 \sin(4t+180^\circ\cdot 9)$
1889, 1890, 1899	$4^\circ\cdot 73 \sin(t+224^\circ\cdot 8)+0^\circ\cdot 79 \sin(2t+42^\circ\cdot 0)$ $+0^\circ\cdot 13 \sin(3t+16^\circ\cdot 1)+0^\circ\cdot 05 \sin(4t+197^\circ\cdot 8)$

Group of years.	Barometric pressure (unit = 0·001 inch).
1892 to 1895	$5\cdot 11 \sin(t+27^\circ\cdot 1)+10\cdot 37 \sin(2t+150^\circ\cdot 9)$ $+0\cdot 57 \sin(3t+3^\circ\cdot 5)+0\cdot 34 \sin(4t+270^\circ\cdot 0)$
1889, 1890, 1899	$3\cdot 99 \sin(t+21^\circ\cdot 7)+10\cdot 78 \sin(2t+148^\circ\cdot 9)$ $+0\cdot 97 \sin(3t+9^\circ\cdot 7)+0\cdot 32 \sin(4t+261^\circ\cdot 4)$

Here again, what strikes one principally is the closeness of agreement between results from periods containing so few years.

§ 63. As supplementing the results for barometric pressure, attention may be drawn to the following important results obtained at the Dutch Colonial Observatory at Batavia* for Fourier series expansions of the diurnal inequality. The unit in all cases is 1 millim. of mercury.

Group of years.	Mean pressure, 758+.	24-hour term.		12-hour term.		8-hour term.	
		Amplitude.	Phase angle.	Amplitude.	Phase angle.	Amplitude.	Phase angle.
1866–70 . . .	0·81	0·61	23°·1	0·94	158°·0	0·04	7°·3
1871–75 . . .	0·43	0·63	25·4	0·95	158·1	0·04	16·6
1876–80 . . .	0·81	0·62	26·4	1·00	160·6	0·04	13·4
1881–85 . . .	0·91	0·66	26·1	1·03	160·5	0·04	20·0
1886–90 . . .	0·71	0·62	28·0	1·03	160·1	0·04	23·4
1891–95 . . .	0·70	0·63	25·7	1·03	161·0	0·04	27·0
1896–1900 . . .	0·82	0·63	25·3	1·01	160·6	0·04	29·6

I would specially call attention to the extreme smallness of the variation in the amplitudes of the several terms, and to the fact that 1891 to 1895 represents, as compared to the two adjacent groups of years, large sun-spot frequency.

We know from the important researches of M. ANGOT, referred to in the "Preliminary Note," that the connection between magnetic variations and sun-spot frequency is unmistakable at Batavia as in Europe.

* 'Observations made at the Mag. and Met. Observatory at Batavia,' vol. 23, 1900, p. 143.

§ 64. In considering results such as those in Table XLVI., allowance must be made for the fact that in the case of such an element as rainfall the results from any single station for a single year, or even occasionally for a group of three years, may not be fairly representative for more than a very limited area of country. But most of the other elements are much more representative in character, and it seems safe to conclude that, as between the two groups ot years of sun-spot maximum and minimum, there was not in the south of England generally any difference of meteorological character at all of the same order as that manifested in the magnetic elements. From this generalization I would however exclude atmospheric electric potential.

There seem, unfortunately, no data whence one can derive trustworthy information as to changes from year to year in the mean potential gradient in the open. An attempt is at present being made at Kew to fill this gap, but with what success remains to be seen. I think information on the point, both at high and low levels, is much to be desired.

§ 65. We shall now consider certain resemblances between magnetic and meteorological phenomena in the average year, which merit consideration in connection with theories as to the source of the magnetic diurnal variations.

If we take the mean monthly values of temperature range and vapour pressure at Kew, used in obtaining Table XLVI., and represent each monthly value as a percentage of the mean of the 12 monthly values, we get the results in Table XLVII. For comparison, this also gives the corresponding monthly percentages which are the means found for the ranges of D and H employed in the calculation of Tables XXXIII. and XXXIV.

Table XLVII.

	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
D and H ranges . . .	54	62	102	128	134	132	131	133	120	96	66	42
Temperature ranges . .	62	75	98	117	125	128	124	116	119	95	76	67
Vapour pressure . . .	72	70	74	85	99	126	139	142	127	102	90	74

In the case of vapour pressure, we are dealing not with a diurnal range, but with the mean value for the 24 hours. I have introduced it principally to show that the seasons of maximum and minimum vapour pressure are conspicuously later in the year than the seasons of maximum and minimum magnetic range, and that the values of vapour pressure in March and December are much alike, whereas magnetic ranges in the former month are double those in the latter.

It will be noticed that the annual range in temperature percentages in Table XLVII., viz., from 62 to 128, is less than that for D and H, and that the winter minimum for temperature occurs unmistakably in January, and so is *later* than the magnetic minimum.

§ 66. The points of agreement and difference between temperature and magnetic phenomena are brought out more fully by a consideration of the Fourier series for the diurnal inequalities of temperature. Such series have been given by General STRACHEY* for first order stations of the Meteorological Office employing an 11-year period, and for Greenwich employing a 20-year period. Taking General STRACHEY'S figures, I have calculated the annual variation of the "c" coefficients—*i.e.*, the amplitudes of the periodic terms—expressing the value for each month as a percentage of the mean of the 12 monthly values. The results are given in Table XLVIII., along with corresponding magnetic data representing the mean results for D, I, H, and V, as given in Tables XXIV. and XXV. In the case of c_1 and c_2, I give a mean result for Valencia, Falmouth, and Stonyhurst, in addition to values for Greenwich and Kew. General STRACHEY'S notation is P for my c.

TABLE XLVIII.—Variation of Fourier Coefficients in the Diurnal Inequality, throughout the Year.

	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
c_1.												
Temperature (Valencia, Falmouth, Stonyhurst)	40	57	97	130	161	149	144	135	115	81	52	40
Temperature, Greenwich	37	62	92	127	145	151	152	141	127	82	55	32
,, Kew	39	52	102	126	147	144	146	139	123	91	54	37
Magnetics, Kew	42	58	91	126	150	147	148	138	122	94	57	27
c_2.												
Temperature (Valencia, Falmouth, Stonyhurst)	91	109	126	114	70	48	58	99	137	137	113	94
Temperature, Greenwich	87	117	125	112	66	52	61	103	158	143	108	70
,, Kew	100	113	157	111	40	33	23	87	154	167	124	90
Magnetics, Kew	56	57	102	134	134	127	130	126	113	94	78	49
c_3.												
Temperature, Greenwich	85	71	43	114	149	144	153	130	69	69	95	78
,, Kew	73	50	37	120	178	134	157	149	102	45	87	68
Magnetics, Kew	54	79	133	140	110	91	94	133	145	119	68	35

§ 67. Before discussing Table XLVIII. I wish to explain my position.

General STRACHEY † has pointed out that the fact that a diurnal inequality can be

* 'Phil. Trans.' for 1893, p. 617.

† *Loc. cit.*, p. 636.

analysed into Fourier terms is no proof of the existence of "distinct physical influences operating in recurring cycles of 24, 12 . . . (hours)." He has in fact shown that the simple hypothesis that the heating effect of the sun varies as the sine of his altitude, being nil when the sun is below the horizon, gives rise to Fourier series which constitute not at all a bad first approximation to the results actually found at certain stations.

Accepting this contention, we may however get much useful information from the FOURIER'S analysis. For instance, we know that the consequences of surface heating on underground temperature can be most easily arrived at by an analysis which treats the Fourier terms as independent entities. Also the examination of differences between the relative values of c coefficients at different seasons of the year, or in different years at the same season, is perhaps the most hopeful way of ascertaining community of origin in different phenomena.

§ 68. Returning to Table XLVIII. we see that in the case of c_1 there is a very close resemblance between the annual variations in temperature and magnetics. The agreement is just about as good as that between the temperature results for Kew and Greenwich, and decidedly better than that between some of the individual magnetic elements at Kew. The difference between December and January is certainly less conspicuous in temperature than in magnetics, but still at both Kew and Greenwich the December temperature mean is the lower.

In the case of c_2 and c_3 the temperature results at the different stations differ more widely than in the case of c_1, but clearly the law of annual variation is here altogether different from that appearing in magnetics. The temperature data for c_4 seemed too small and uncertain to be worth detailed consideration.

As regards the relative importance of the different Fourier terms in temperature and magnetics, the following are the ratios for the yearly means :—

	c_2/c_1.	c_3/c_1.	c_4/c_1.
Temperature (after STRACHEY) mean from Greenwich . . .	·21	·08	·03
" " " 7 British stations .	·23	·10	—
Magnetics (at Kew) mean from D, I, H and V	·64	·32	·13

This shows the much greater relative importance of the 24-hour term in temperature than in magnetics. The difference is probably even greater than the figures suggest. The magnetic values of c_1, &c., refer to mean inequalities for the year; whereas General STRACHEY'S corresponding values represent arithmetic means of 12 monthly values, and his phase angles are most variable in the shorter period terms. It will be seen in fact that the values obtained for c_2/c_1, &c., from the mean temperature inequalities for 1892 to 1895, or 1889, 1890, and 1899, in § 62, are less than the values we have just derived from General STRACHEY'S figures.

§ 69. A rather striking illustration of the points of agreement and difference just discussed, was afforded by the case of V at Kew. As already stated, all the tables had to be re-calculated, making allowance for the non-application of a temperature correction in earlier years of the period. I was puzzled for a time by the, at first sight, remarkable fact that whilst the new values of c_1 differed very appreciably from the old, there was exceedingly little difference between the new and old values of the other c coefficients, or of any of the phase angles. The explanation simply was that the external diurnal temperature inequality was pretty exactly represented in the magnetograph room—of course, on a very much reduced scale—with the hours of maximum and minimum so retarded as to bring them much closer to the corresponding times for the diurnal variation of V.

§ 70. There are other important conclusions to be drawn from the comparison of temperature and magnetics. The source of the solar diurnal inequality in magnetism is, of course, the sun, acting in some way or other; the only question is how. The researches of GAUSS, SCHMIDT, ADAMS, and others all point to the conclusion that magnetic force at the Earth's surface is derived, if not exclusively, at least in a wholly preponderating degree, from centres of force within that surface. Accepting this conclusion, one would most naturally perhaps look for the action of the sun on the Earth itself as the cause of the diurnal inequality. MAXWELL and others have pointed out that the tidal effect of the sun by modifying internal strains must exert *some* influence on magnetic strata; but the trifling size of the lunar magnetic inequality indicates that any such effect must be very small.

The only other obvious direct solar effect on the Earth itself is the diurnal heating and cooling of the surface. It has been pointed out, however, by Principal RÜCKER that, owing to the great preponderance of ocean, the portion of the Earth's surface exposed to appreciable temperature variation is comparatively small. What I wish to point out in addition to this is that any diurnal variation of temperature in the Earth diminishes very rapidly as the depth increases, and the rate of decrease is more rapid for the part of the surface heating represented by the 12-hour and 8-hour Fourier terms than for the part answering to the 24-hour term. Also any seasonal underground change of temperature lags behind the surface change. Now we have seen that the ratios borne by the amplitudes of the higher order Fourier terms to the amplitude of the 24-hour term are not smaller, but much larger, in the magnetic elements than in temperature; whilst magnetic seasonal changes seem earlier than those of surface temperature, and much earlier than those presented by an element such as vapour pressure known to be directly dependent on temperature. There is, of course, the further obvious point that surface temperature shows no large difference from years of sun-spot maximum to years of minimum, and still less will underground temperature.

This reasoning may appear quite unnecessary to those who accept Professor SCHUSTER's able paper in the 'Phil. Trans.' for 1889, p. 467, as *proving* the chief if not exclusive source of the magnetic diurnal inequality to be external to the Earth. I must confess, however, that whilst I consider the conclusion, which has been arrived at by other physicists from other points of view, to be probably in the main correct, I think that the investigation ought to be repeated when practicable for at least two different groups of years, one representing numerous, the other few sun-spots; and trustworthy *simultaneous* data from at least ten or twelve well distributed observatories would be highly desirable, if not absolutely necessary. Meantime, I suspect, the vertical force is likely to prove a stumbling block.

Nature of the Relationship between Sun-Spots and Terrestrial Magnetism.

§ 71. It was pointed out in the "Preliminary Note" that WOLFER's table of sun-spot frequencies is based on *all* days for which solar observations exist, whereas the Kew magnetic data refer to five days a month only. If one divided the days of a month for which sun-spot data exist into two equal groups, the one containing the days of the lowest frequency, and if from this group one selected five days at random, the mean frequency for these days would often be but a small fraction of WOLFER's own mean for the month. Thus if, as has frequently been suggested, sun-spots were the immediate cause of magnetic disturbances, the disturbance being synchronous with the spot, the presumption would be that the sun-spot frequency for *all* and for *quiet* days would be altogether different, the latter being much the smaller. As explained in the "Note," data which WOLFER does publish for individual days were not used for the following reasons; (1) being only *provisional* they are presumably less trustworthy than the finally accepted monthly values appearing in Table I.; (2) not infrequently data were wanting for several of the selected quiet days.

I have, however, looked carefully into the matter, to see whether the results here arrived at would have been materially modified if sun-spot frequencies had been taken from *quiet* days only. For this purpose, frequencies were calculated for each month, using WOLFER's *provisional* values for the selected *quiet* days. These are compared in Table XLIX. with the corresponding data derived by WOLFER from his provisional values for *all* days. In forming the means I have left out of account three months when there were less than two quiet days.

TABLE XLIX.—WOLFER's *Provisional* Sun-Spot Frequencies.

	All days.	*Quiet* days.		*All* days.	*Quiet* days.		*All* days.	*Quiet* days.
January . .	40·1	47·6	May . . .	43·5	42·2	September .	45·6	45·7
February. .	40·4	42·0	June . . .	45·3	43·3	October . .	41·6	41·1
March . .	35·6	30·0	July . . .	45·4	42·3	November .	32·4	35·1
April . . .	41·2	39·8	August . .	45·7	47·4	December .	35·6	37·3

I also show side by side in Table L. the mean seasonal and yearly values obtained for sun-spot frequencies from Table I. and from the two sets of provisional data made use of in Table XLIX. Results are given for the groups of sun-spot maximum and minimum years, as well as for the whole period.

TABLE L.—Mean Seasonal and Yearly Values of WOLFER's Sun-Spot Frequencies.

	Winter.			Equinox.			Summer.			Year.		
	11 years.	1892–5.	1890, &c.	11 years.	1892–5.	1890, &c.	11 years.	1892–5.	1890, &c.	11 years.	1892–5.	1890, &c.
Final values Table I.	39·4	71·1	8·2	41·0	68·9	11·2	44·7	84·8	9·2	41·72	75·0	9·5
Provisional *all* days	37·4	67·1	6·9	41·0	69·2	11·3	45·0	85·6	9·2	41·22	74·1	9·1
Provisional *quiet* days.	40·8	72·7	8·7	39·2	69·0	10·6	43·8	81·8	10·4	41·28	74·5	9·9

§ 72. The smallness of the differences between the means from *all* and from *quiet* days seems to dispose absolutely of any theory which regards sun-spot frequency on a given day as any guide whatsoever to the quiet or disturbed character of Terrestrial Magnetism on that particular day. For our immediate purpose, however, the important thing is the evidence afforded by the above Tables, that the results we have obtained for the a and b constants in the sun-spot formula, would be but little affected if we took frequencies answering to magnetically quiet days only.

In the methods in which we have employed groups of years, the sun-spot data required for calculating seasonal or mean yearly values of b and a are those given in Table L. It is thus obvious, so far at least as mean annual values are concerned, that the result of substituting *quiet* day frequencies would be as nearly as possible nil. Summer values of b and b/a would be slightly raised by the substitution, winter and equinoctial values very slightly diminished.

§ 73. I next investigated whether rises and falls in WOLFER's frequencies, as given in Table I., and as deduced from provisional values for quiet days proceeded on parallel lines. Table I. presents a rather singular feature. If we attach the signs + or − according as there is a rise or fall in the frequency in passing from one month to the next, we find that there are no fewer than 90 changes of sign out of a total possible of 130. We have long sequences of + and − occurring alternately, *e.g.*, from January, 1895, to March, 1896. It looks as if the monthly changes generally went beyond the temporary equilibrium position, or else as if the estimate of them generally overshot the mark. The quiet day provisional frequencies present the same feature, though the monthly changes they show differ appreciably in general from those deduced from Table I. Omitting the three months already referred to, we have 125 monthly changes common to the two sets of frequencies. Of these 97 agree, and 28 differ in sign. If, however, we take only those cases in which the monthly change amounted to at least 10 in one or other of the two sets of data, we find an agreement in sign in no less than 60 cases out of 72.

Passing to the monthly changes in the percentage values of the D and H ranges, given in Tables XXXIII. and XXXIV., we find there also frequent alternations of rises and falls. For instance, in Table XXXIII., rises and falls alternate without a break from January, 1893, to January, 1894. On the whole, however, there are appreciably fewer transitions of sign in the D and H Tables than in Table I. When the monthly changes in the two sets of sun-spot frequencies differ in sign, the changes in Tables XXXIII. and XXXIV. agree sometimes with the one set, sometimes with the other, there being in both tables a very slight preponderance of agreements with WOLFER's *all* day frequencies. This may be purely accidental, but at all events the magnetic changes agree at least as well with the sun-spot data from all days as with those from quiet days only.

§ 74. There still remains the question whether there is a distinct connection between the mean values for individual *months* of sun-spot frequencies and magnetic ranges. In the case of H there does seem fairly definite evidence of such a connection. If we take the changes of sign we have just been considering in Tables I. and XXXIV., we find an agreement in sign in 74 cases, as against disagreement in 55 cases; and if we confine ourselves to cases where the changes between successive months' values, in Table I., is at least 10, we find agreement in 34 cases out of 50. This amount of agreement is more than would be at all likely to happen by pure chance. In the case of D, however, the agreements and disagreements in sign are almost equally numerous, and even when we confine ourselves to cases where the sun-spot change was at least 10 the balance of agreements is too small to be relied on.

In the case of H a diurnal range of 50γ is exceptionally large; the curve is read only to 1γ; the average non-cyclic effect is 3γ, and there may be a small uncertainty through temperature. Thus the uncertainty in individual figures in Table XXXIV. cannot well be less than 2 or 3 per cent. even at midsummer. In the case of D, the range often exceeds $10'$, and seldom falls below $5'$; the curve is read to $0'\cdot1$; the non-cyclic effect is very small, and there is no temperature correction. Thus the uncertainty in individual figures in Table XXXIII. should be considerably less than in Table XXXIV. Thus the absence of an unmistakable connection between D ranges and sun-spot frequencies in individual months has stronger evidence in its favour than has the apparent connection between H ranges and monthly sun-spot frequencies.

If instead of individual months we take years the connection is clear, and it is manifest enough, even in D, for considerably shorter periods than years at times when sun-spot frequency has a rapid general drift in one direction.

§ 75. Some light on the degree of intimacy of the connection is derivable from the amount of agreement between the individual monthly ranges observed in D, H and I and those calculated from the values of a and b in Table XL. The probable errors calculated from the differences between the eleven observed and calculated values for four months representing different seasons of the year were as follows:—

	D.	I.	H.
Greatest	0'·9	0'·23	3·2γ
Least	0·5	0·13	2·2γ
Mean	0·65	0·18	2·8γ

These probable errors are by no means large, considering that the range for any individual month depends on only five days' results, and that the non-cyclic effects in I and H are so considerable. Still the results tend to confirm the conclusion to which other considerations point, that the departures from the mean shown by magnetic phenomena on quiet days cannot be solely determined by the simultaneously existing sun-spot frequency, unless we are prepared to hold that either WOLFER's sun-spot data or Kew magnetic data are habitually affected by considerable errors.

§ 76. There are a variety of ways of explaining the phenomena, some of which may be indicated briefly. The whole solar system may be under the action of some external agency, whose effect on the sun is made manifest by the occurrence of sun-spots. As the influence simultaneously existent would naturally vary from part to part of the solar system, this hypothesis would not be inconsistent with the view that the influence on the sun itself is measured by the sun-spot frequency. The influence might be of the nature of a radiation making the Earth's atmosphere offer a lessened resistance to the electric currents due to solar action to which various physicists ascribe the diurnal magnetic inequality.

Or the sun itself might be the sole agent, if we suppose that sun-spot frequency is a qualitative rather than a quantitative measure of its activity. The sun-spot might stand to the really active cause somewhat as the smoke from a locomotive to the heat of its furnace. When smoke issues we know the fire is alight, but we cannot deduce the actual heat.

There is a third obvious alternative, viz., that the sun-spot frequency is a direct measure of the sun's contemporaneous activity, but that the effect at the earth depends appreciably on what has been happening at the sun for some time previously. This is what we might expect to happen if the sun were the origin of a radiation which took a considerable time to part with all its ionising power, or which travelled so much slower than light that emanations leaving different parts of the sun simultaneously reached the earth at appreciably different times. If however this third alternative were true, we should expect magnetic phenomena to show a general tendency to lag behind sun-spot frequency. Mr. ELLIS has made investigations on this point, but has detected no certain lag. It is, however, open to doubt whether a short lag—a month or two for instance—would with certainty be shown by the method he employed.

Before concluding, I wish to acknowledge the valuable assistance given me by several members of the staff of the Observatory Department of the National Physical Laboratory.

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To Mr. G. BADDERLY and Mr. B. FRANCIS I am indebted for much careful arithmetical work, especially in connection with the calculation of Fourier coefficients.

The calculation work has been exceedingly heavy, and some errors have doubtless escaped me, but every reasonable precaution has been taken to secure accuracy.